CWA extensions to Multi-Valued Logics

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Abstract. Various CWA formalizations including the naive CWA, the generalized CWA and the careful CWA are extended to multi-valued logics in a uniform manner by using the so-called polynomial forms of clauses.

1 Introduction

In the area of deductive databases, people usually judge whether a proposed declarative semantics is successful according to three criteria. The first one concerns its ability to reflect the common sense or intended meaning of databases. The second one concerns the computational issue. And the third one is obviously the expressive power of the language characterized by the semantics[3].

This papers is motivated by carrying out these criteria. First of all, to enhance the expressive power of databases(Criterion 3), we see that it is quite meaningful to build up deductive database techniques in some non-classical logic framework. As a matter of fact, in recent decades a lot of work has been done along this line(e.g. [2]).

Multi-valued logics are those where the propositions can have other values than the usual true and false values. They are very useful from both theoretical and practical points of view. Some questions are better explained with the consideration of such logics. For instance, we can thus cope with the databases that involve uncertainty and imprecise information, etc. For these we refer the reader to Chapter 7 of [11].

In this paper we focus on multi-valued logic databases. we extend various CWA formalizations to multi-valued logics The reason is that, as in the classical case, the usual inference rules are not sufficient anymore to catch the common sense or intended meaning of databases. The introduction of such formalizations may clearly increase their declarative ability(Criterion 1).

On the other hand, as is known, a database is usually so large that its explicit representation is practically expansive or impossible. One has to establish inference rules by which some information can be inferred implicitly. So it's natural to think of whether the CWA formalizations still work in multi-valued logics.

We discuss in an algebraic framework[12][13][5] by embedding logic formulas in some polynomial ring. The related problems are therefore translated into algebraic ones. For example, loosely speaking the provability and consistency problems correspond to the polynomial ideal membership problem. Such algebraic formulations can be studied by well-known symbolic computation systems such as MAPLE and MATHEMATICA. Furthermore, the CWA formalizations presented in this paper can be applied in a uniform manner to any finitely multi-valued logics. So they have not only a theoretical but a computational interest(Criterion 3).

Historical remark. The (naive) CWA in classical two-valued logic was introduced by Reiter[15]. It provides an inference rule to deal with the negative information from databases. If it is in force, one needn't represent the whole relevant information. However, CWA may be inconsistent even though the original database is consistent. So sometimes this formalism is both inappropriate and unsafe. The generalized CWA was thus proposed by Minker[14] to overcome this problem. And later on, Gelfond *et al* developed two more powerful formalizations to enhance the declarative ability of the generalized CWA, which is referred to as the careful CWA, and the extended CWA[8][9][10].

In this paper we extend the CWA, the generalized CWA as well as the careful CWA to any finitely multi-valued logics. We also present an example to demonstrate that the extended CWA can by no means be generalized following our treatment.

2 Basic knowledge: Multi-valued logics

We are interested in finitely multi-valued logics. The reader is referred to [4] for the notions such as *atoms*, *formulas*, *ground formulas*, *interpretations* and *models* of a formula set.

In this paper a *clause* is defined to be a closed formula of the form $(\forall x_1)\cdots(\forall x_k)\varphi$, where φ is a formula containing no quantifiers. We always omit the universal quantifiers in a clause without confusion.

Throughout the paper we take a finite rational number set $V = \{0, 1/(m-1), \cdots, (m-2)/(m-1), 1\}(m \geq 2)$ from the interval $[0, 1]$ to denote the set of *truth values*, and $T = \{k/(m-1), \cdots, (m-2)/(m-1), 1\} \subseteq V (k \leq m-1)$ the *designated truth values*. Let $F = V - T$.

As usual we suppose that there are no other function symbols than constants. We always use S to denote a clause set. The *Herbrand universe* H, *Herbrand base* HB, *Herbrand interpretation* as well as *Herbrand model* are defined similarly as in the classical two-valued logic. Then, S has a model iff S has a Herbrand model. A Herbrand interpretation is often represented by the set $I = \{p = v \mid p \in HB, v \in V\}$. By $S \models \varphi$ we mean that all the Herbrand model of S are models of the formula φ.

3 Translating clauses into polynomial forms

Now, we construct a set PS of polynomial forms from S such that all the Herbrand models of S can be seen as the zeros of PS. We then consider the consistency and provability problems in the polynomial language.

3.1 Polynomial forms

For the concepts in commutative algebra that are used but not defined in this paper, for example, *polynomial*, *polynomial ring*, *ideal*, *radical* and *zero set*, the reader consults [6]. Now let Q be the rational number field. A polynomial over Q with atoms as indeterminates is called a *polynomial form.*

For each n-place connective ϕ, we associate a polynomial P_ϕ in the polynomial ring $Q[x_1, \cdots, x_n]$:

$$P_\phi(x_1, \cdots, x_n) = \sum_{v_1, \cdots, v_n \in V} \phi(v_1, \cdots, v_n) \prod_{i=1}^{n} \prod_{u \in V, u \neq v_i} \frac{x_i - u}{v_i - u}.$$

In fact, P_ϕ is obtained by using the Lagrange interpolation formula. For any $(v_1, \cdots, v_n) \in V^n$, $P_\phi(v_1, \cdots, v_n) = \phi(v_1, \cdots, v_n)$.

For $\varphi \in S$, we call the $P\varphi$ defined below the *polynomial form of* φ:

$$P\varphi = \begin{cases} \varphi, \text{if } \varphi \text{ is an atom;} \\ P_\phi(P\varphi_1, \cdots, P\varphi_n), \text{if } \varphi = \phi(\varphi_1, \cdots, \varphi_n), \end{cases}$$

where ϕ is an n-place connective, $\varphi_1, \cdots, \varphi_n$ are clauses.

For the limitation of space, we no longer provide examples to illustrate how to obtain the polynomial form of a clause. For details the interested reader may consult [16].

Now, let $Q[HB]$ be the polynomial ring over Q with the elements of HB as indeterminates. Unless stated otherwise, in the sequel all the polynomials and ideals mentioned are in $Q[HB]$. For polynomial sets $J_i (i = 1, \cdots, k)$, we use $(J_1, \cdots, J_k)$ and $Zero(J_1, \cdots, J_k)$ to denote the ideal generated by and the zero set (in the complex number field) of $\cup_{i=1}^{k} J_i$, respectively. By $\sqrt{J}$ we denote the radical of ideal J.

Let $PS = \{\prod_{v \in T}(v - P\varphi) \mid \varphi \in S\}$. For a polynomial form ψ, the polynomial form obtained by replacing each variable in ψ by an element of H is called a *ground instance* of ψ. Let $PS|H$ be the set of all ground instances of the members of PS, $AHB = \{\prod_{v \in V}(p - v) \mid p \in HB\} \subseteq Q[HB]$, and $NF \subseteq Q[HB]$. We call the elements of $Zero(PS|H, NF, AHB)$ the *Herbrand models* of $PS \cup NF$. We then have

Theorem 3.1 I is a Herbrand model of S iff I is a Herbrand model of PS.

This theorem actually indicates that the notion of Herbrand models of a corresponding polynomial form set is identical to the usual one. Similarly, if $\psi \in Q[HB]$ comes from a ground clause φ, then the zeros of ψ are models of φ. So we often call a zero of ψ a model of ψ.

3.2 Consistency and provability

To introduce various CWA formalizations into multi-valued logics, we first have to investigate the consistency and provability problems in the polynomial language.

$PS \cup NF$ is said to be *consistent* if $(PS|H, NF, AHB) \neq (1)$, where (1) is the trivial ideal $Q[HB]$. For $\psi \in Q[HB]$, we say that ψ is *provable from* $PS \cup NF$ ($PS \cup NF \vdash \psi$) if $\psi \in \sqrt{(PS|H, NF, AHB)}$. By Hilbert Nullstellensatz[6] and Theorem 3.1, we have

Theorem 3.2 (1) PS is consistent iff S is consistent; (2) Let φ be a ground clause. $PS \vdash \prod_{v \in T}(v - P\varphi)$ iff $S \models \varphi$.

Theorem 3.2(1) shows that the consistency problem is indeed identical to the trivial ideal problem. We can thus decide this problem by using the Groebner basis method. In fact, $(PS|H, NF, AHB) = (1)$ iff the reduced Groebner basis of $(PS|H, NF, AHB)$ under a fixed monomial order is $\{1\}$.

In practice, one usually uses the Herbrand universe and the Herbrand models. Theorem 3.1(2) demonstrates that the provability problem in

Herbrand model sense is just the radical membership problem. We can decide it using the Groebner basis method as well. Actually,

$$\psi \in \sqrt{(PS|H, NF, AHB)} \text{ iff } (PS|H, NF, AHB, \{1 - z\psi\}) = (1),$$

where z is a new indeterminate, $(PS|H, NF, AHB, \{1 - z\psi\})$ is in the polynomial ring over Q with the elements of HB and z as indeterminates.

For the details about how to compute the Groebner bases, please see [6]. Note that we can also use the characteristic set method to solve these problems[16] [17].

4 CWA in multi-valued logics

In this section, we define the CWA in multi-valued logics by employing polynomial forms, investigate the logic meaning of the newly-added polynomial forms, and then discuss the query evaluation under the CWA.

4.1 Syntax and semantics

We propose the following syntactic formalization of the CWA in multi-valued logics. Let

$$NF = \{\prod_{v \in F}(p - v) \mid p \in HB, PS \not\vdash \prod_{v \in T}(v - p)\}, CWA(S) = PS \cup NF.$$

For a ground clause φ, if $CWA(S) \vdash \prod_{v \in T}(v - P\varphi)$, we say that φ is *provable from S under the CWA*.

Adding an element of NF, say $\prod_{v \in F}(p - v)$, to PS can be seen as a further restriction on p than $\prod_{v \in V}(p - v) \in AHB$. In classical two-valued logic case, this definition has no difference from the usual one. Accordingly we have

Theorem 4.1 Assume that S is consistent, and $p \in HB$. (1) $CWA(S) \vdash \prod_{v \in T}(v - p)$ iff $PS \vdash \prod_{v \in T}(v - p)$; (2) $CWA(S) \vdash \prod_{v \in F}(p - v)$ iff $PS \not\vdash \prod_{v \in T}(v - p)$.

This theorem says that no new ground atoms are provable from S under the CWA if $CWA(S)$ is consistent. Moreover, when $\prod_{v \in F}(p - v)$ is viewed as the negation of p, then if a ground atom is not provable from S, its negation is provable from S under the CWA.

Now we want to provide a semantics for $CWA(S)$. Let HM be the set of all Herbrand models of S, and for $I \in HM$, $I_T = \{p \in HB \mid \text{there exists } t \in T, p = t \in I\}$. We define a binary relation Ξ on HM:

$$\text{For } I, I' \in HM, I \Xi I' \text{ iff } I_T = I'_T.$$

Ξ is an equivalence on HM. Let $[I] = \{I' \in HM \mid I \Xi I'\}, HM/\Xi = \{[I] \mid I \in HM\}$. We define a binary relation $\preceq$ on HM/Ξ:

$$\text{For } I, I' \in HM, [I] \preceq [I'] \text{ iff } I_T \subseteq I'_T.$$

It isn't hard to examine that $\preceq$ is a partial order on HM/Ξ.

For $I \in HM$, if for any $I' \in HM$, $[I'] \preceq [I]$ implies $[I'] = [I]$, then $[I]$ is called a *minimal Herbrand model set* of S. If $[I]$ is the unique minimal Herbrand model set of S, we call $[I]$ the *least Herbrand model set* of S.

The notion of least Herbrand model set provides a semantic characterization of the consistency of $CWA(S)$. And the minimal Herbrand model sets can be used to describe the semantics of the CWA. In fact, by Theorems 3.1 and 4.1 we have the following two theorems.

Theorem 4.2 $CWA(S)$ is consistent iff S has the least Herbrand model set.

In classical two-valued logic, Ξ is the identity relation on HM. Each equivalence class over HM determined by Ξ consists of only one Herbrand model of S. So in this case Theorem 4.2 coincides with the conclusion that $CWA(S)$ is consistent iff S has the least Herbrand model.

Theorem 4.3 If $CWA(S)$ is consistent, then for any $\psi \in Q[HB]$, $CWA(S) \vdash \psi$ iff all members of the least Herbrand model set of S are models of ψ.

Therefore, for any ground clause φ, if $CWA(S)$ is consistent, then φ is provable from S under the CWA iff all the members of the least Herbrand model set of S are models of φ.

4.2 Logic correspondent to NF

In the alphabet of the underlying multi-valued, say m-valued, logic, we add, if there doesn't exist, an 1-place connective $\diamond$, which satisfies the following condition:

$$\text{If } v \in F \text{ then } \diamond(v) \in T; \text{ Otherwise } \diamond(v) \in F.$$

We call $\diamond$ a *weak negation*. If F contains k elements, then there exist obviously $k^{(m-k)}(m-k)^k$ weak negations.

In classical two-valued logic, there is only one weak negation, namely the regular negation. In Kleene's three-valued logic[4], there are two weak negations. For example, the following $\sim$ is one of them: $\sim(0) = 1$, $\sim(1/2) = 1$, and $\sim(1) = 0$. As a matter of fact, $\sim$ was introduced in [7], it is very useful in modeling a non-monotonic logic by utilizing a three-valued logic.

Let $\diamond$ be a weak negation, and $NFS = \{\diamond p \mid p \in HB, S \not\models p\}$. By Theorem 3.1 and 4.1(2), we have

Theorem 4.4 I is a Herbrand model of $PS \cup NF$ iff I is a Herbrand model of $S \cup NFS$.

Hence, adding NF to PS is just identical to adding NFS to S. We may thus define $CWA(S)$ to be the set $S \cup NFS$. Remark that it doesn't matter which weak negation is chosen. However, a number of multi-valued logics, for example, Kleene's three-valued logic, are not functionally complete. So the weak negations may not be represented by the connectives in the underlying multi-valued logics.

4.3 Query evaluation under the CWA

Let $\varphi(y_1, \cdots, y_r, x_1, \cdots, x_s)$ be a quantifier-free formula, in which no function symbols occur, and $y_1, \cdots, y_r, x_1, \cdots, x_s$ are all the variables appear.

$$Q =< y_1, \cdots, y_r \mid (\exists x_1) \cdots (\exists x_s)\varphi(y_1, \cdots, y_r, x_1, \cdots, x_s) >$$

is called a *query*. If $\varphi(y_1, \cdots, y_r, x_1, \cdots, x_s)$ is an atom, then Q is said to be an *atomic query*.

$A \subseteq H^r$ is called an *answer* to Q (w.r.t. S) if for an arbitrary Herbrand model I of S, there exists $(c_1, \cdots, c_r) \in A$ such that I is a model of the formula $(\exists x_1) \cdots (\exists x_s)\varphi(c_1, \cdots, c_r, x_1, \cdots, x_s)$. An answer A to Q is said to be *minimal* if any proper subsets of A are not answers to Q. If A is a minimal answer to Q, and consists of only one element, then A is called a *definite answer* to Q. We use $||Q||$ to denote the set of all the minimal answers to Q. Let

$$\psi = \prod_{(c_1,\cdots,c_r)\in A} \prod_{(h_1,\cdots,h_s)\in H^s} \prod_{v\in T} (v - P\varphi(c_1, \cdots, c_r, h_1, \cdots, h_s)).$$

Then A is an answer to Q iff $PS \vdash \psi$. We call A a *CWA answer* to Q (w.r.t. S) if $CWA(S) \vdash \psi$. The *minimal* and *definite CWA answers* to Q

can be defined similarly. In the sequel, by $||Q||_{CWA}$ we denote the set of all minimal CWA answers to Q. We then have

Theorem 4.5 Suppose that $CWA(S)$ is consistent. (1) If there is only one element in the least Herbrand model set of S, then for an arbitrary query Q, every minimal CWA answer to Q is definite; (2) For any an atomic query Q, every minimal CWA answer to Q is definite.

It's shown in [15] that all minimal CWA answers to Q are definite for any query Q in classical two-valued logic. It is in fact a corollary of Theorem 4.5(1). This conclusion doesn't hold anymore in some multi-valued logics including Kleene's three-valued logic, However, Theorem 4.5(2) says it is the case for atomic queries.

We know that for any atomic query Q, $||Q|| = ||Q||_{CWA}$ in classical two-valued logic[15]. The following theorem shows that it is still true in any multi-valued logics. So, we needn't know the elements of NF for atomic query evaluation. Here we omit the proof since it is straightforward.

Theorem 4.6 Suppose that $CWA(S)$ is consistent. If Q is an atomic query, then $||Q|| = ||Q||_{CWA}$.

5 Other CWA formalizations in multi-valued logics

Hereafter, we discuss the generalized CWA and careful CWA in multi-valued logics. We begin with the careful CWA because the former is indeed a special case of the latter. We also show that the extended CWA cannot be generalized to multi-valued logics following our treatment.

5.1 Careful CWA

For any subset G of the set of all predicate symbols appearing in S, let

$$G^+ = \{\prod_{v\in T}(v-p) \mid p \in HB, \text{ and the predicate symbol in } p \text{ is in } G\}.$$

$$G^- = \{\prod_{v\in F}(p-v) \mid p \in HB, \text{ and the predicate symbol in } p \text{ is in } G\}.$$

In the sequel, the set of all the predicate symbols occurring in S is divided into three disjoint subsets: A, B and C. We first provide a syntactic specification of the Careful CWA.

We define $F(A,B,C)$ to be the subset of HB: For $p \in HB$, $p \in F(A,B,C)$ iff $\prod_{v\in T}(v-p) \in A^+$, and there are no $\psi_1, \cdots, \psi_k \in A^+ \cup C^+ \cup C^- (k \geq 0)$, such that $PS \vdash \psi_1 \cdots \psi_k \prod_{v\in T}(v-p)$ and $PS \not\vdash \psi_1 \cdots \psi_k$. Let

$$NF = \{\prod_{v\in F}(p-v) \mid p \in F(A,B,C)\}, CCWA(S;A,B,C) = PS \cup NF.$$

We say that a ground clause φ is *provable from S under the careful CWA w.r.t.* (A,B,C) if $CCWA(S;A,B,C) \vdash \prod_{v\in T}(v - P\varphi)$. In classical two-valued logic this definition coincides with the usual one. According to this definition we have

Theorem 5.1 For $\psi_1, \cdots, \psi_k \in A^+ \cup C^+ \cup C^-$, $CCWA(S;A,B,C) \vdash \psi_1 \cdots \psi_k$ iff $PS \vdash \psi_1 \cdots \psi_k$.

This theorem provides some clues on the power of careful CWA. It shows that if a new ground atom is provable from S under the careful CWA w.r.t. (A,B,C), then its predicate symbol must be contained in the subset of varying predicate symbols. Also by this theorem we can prove the following theorem, which indicates that $CCWA(S;A,B,C)$ preserves consistency.

Theorem 5.2 $CCWA(S;A,B,C)$ is consistent iff S is consistent.

Now we consider the semantics for the Careful CWA. Suppose that HM, I, I_T are those defined in Section 4.1. In the following, for a subset G of the set of all the predicate symbols appearing in S, we use $I_T|G|$ to represent the subset of I_T:

$$\text{For } p \in I_T, p \in I_T|G| \text{ iff } \prod_{v\in T}(v-p) \in G^+.$$

We first define a binary relation $\Xi_{(A,B,C)}$ on HM:

$$\text{For } I, I' \in HM, I\Xi_{(A,B,C)}I' \text{ iff } I_T|A| = I'_T|A|, \text{ and } I_T|C| = I'_T|C|.$$

$\Xi_{(A,B,C)}$ is an equivalence on HM. Let

$$[I] = \{I' \in HM \mid I\Xi_{(A,B,C)}I'\}, HM/\Xi_{(A,B,C)} = \{[I] \mid I \in HM\}.$$

Now we define a binary relation $\preceq_{(A,B,C)}$ on $HM/\Xi_{(A,B,C)}$:

$$\text{For } I, I' \in HM, [I] \preceq_{(A,B,C)} [I'] \text{ iff } I_T|A| \subseteq I'_T|A| \text{ and } I_T|C| = I'_T|C|.$$

$\preceq_{(A,B,C)}$ is a partial order on $HM/\Xi_{(A,B,C)}$. For $I \in HM$, if for any $I' \in HM$, $[I'] \preceq_{(A,B,C)} [I]$ implies $[I'] = [I]$, then $[I]$ is called an *(A, B, C)-minimal Herbrand model set* of S. Accordingly we can prove

Lemma 5.3 Suppose $\psi_1, \cdots, \psi_k \in A^+ \cup C^+ \cup C^-$. If all the elements of the (A, B, C) -minimal Herbrand model sets of S are models of $\psi_1 \cdots \psi_k$, then $S \vdash \psi_1 \cdots \psi_k$.

Lemma 5.4 Let I be an element of an (A, B, C)-minimal Herbrand model set of S, and $p \in HB$, whose predicate symbol is not in B. Suppose that I is a model of p. For each Herbrand model I' of S that is not a model of p, there exists a $\psi_{I'} \in A^+ \cup C^+ \cup C^-$, such that I' is a model of $\psi_{I'}$ and I is not a model of $\psi_{I'}$.

By Theorems 3.1, 3.2(1), 5.1 and the above two lemmas, we have

Theorem 5.5 Suppose $p \in HB$, and $\prod_{v \in T}(v - p) \in A^+$. $\prod_{v \in F}(p - v) \in NF$ iff all elements of the (A, B, C)-minimal Herbrand model sets of S are models of $\prod_{v \in F}(p - v)$.

Theorem 5.6 For $\psi \in A^+ \cup A^- \cup C^+ \cup C^-$, $CCWA(S; A, B, C) \vdash \psi$ iff all elements of the (A, B, C)-minimal Herbrand model sets of S are models of ψ.

Therefore, for a ground atom whose predicate symbol is in A or C, it is provable from S under the careful CWA w.r.t. (A, B, C) iff all elements of the (A, B, C)-minimal Herbrand model sets of S are its models.

Theorem 5.6 doesn't hold for an arbitrary $\psi \in Q[HB]$. It cannot be even extended for $\psi \in B^+ \cup B^-$. In addition, Theorem 5.2 can be treated as a corollary of Theorem 5.6.

Remark that we can also introduce a two-place connective, say $\sqcup$, which satisfies the condition $\sqcup(v_1, v_2) \in T$ iff $v_1 \in T$ or $v_2 \in T$. Using this two-place connective and the weak negation introduced in Section 4.2, we may define $CCWA(S)$ similarly. In classical two-valued logic and Kleene's three-valued logic, the connective $\vee$ satisfies the condition. So we may choose $\vee$ to be $\sqcup$ in these two cases.

5.2 Generalized CWA

Let F be the subset of HB: For $p \in HB, p \in F$ iff there are no $p_1, \cdots, p_k \in HB(k \geq 0)$, such that

$$PS \vdash (\prod_{v \in T}(v - p)) \prod_{i=1}^{k} \prod_{v \in T}(v - p_i), \text{ and } PS \nvdash \prod_{i=1}^{k} \prod_{v \in T}(v - p_i).$$

Let $NF = \{\prod_{v \in F}(p - v) \mid p \in F\}, GCWA(S) = PS \cup NF$. Clearly $GCWA(S) \subseteq CWA(S)$; If $CWA(S)$ is consistent, then $GCWA(S) = CWA(S)$. We say that a ground clause φ is *provable from S under the generalized CWA* if $GCWA(S) \vdash \prod_{v \in T}(v - P\varphi)$. In classical two-valued logic this is just the usual definition of generalized CWA.

If A is chosen as the set of all predicate symbols occurring in S and $B = C = \emptyset$ in the careful CWA, then $GCWA(S) = CCWA(S; A, B, C)$. So, as corollaries of Theorem 5.1 and Theorem 5.2, we have the following two theorems, which show that $GCWA(S)$ preserves consistency, and no new ground atoms are provable from S under the generalized CWA.

Theorem 5.7 For $p_1, \cdots, p_k \in HB$, $GCWA(S) \vdash \prod_{i=1}^{k} \prod_{v \in T}(v - p_i)$ iff $S \vdash \prod_{i=1}^{k} \prod_{v \in T}(v - p_i)$.

Theorem 5.8 $GCWA(S)$ is consistent iff S is consistent.

The following two theorems are corollaries of Theorem 5.5 and Theorem 5.6, respectively. A ground atom is provable from S under the generalized CWA iff all elements of the minimal Herbrand model sets of S are its models.

Theorem 5.9 For $p \in HB$, $\prod_{v \in F}(p - v) \in NF$ iff all elements of the minimal Herbrand model sets of S are models of $\prod_{v \in F}(p - v)$.

Theorem 5.10 For $p \in HB$, (1) $GCWA(S) \vdash \prod_{v \in T}(v - p)$ iff all elements of the minimal Herbrand model sets of S are models of $\prod_{v \in T}(v - p)$; (2) $GCWA(S) \vdash \prod_{v \in F}(p - v)$ iff all elements of the minimal Herbrand model sets of S are models of $\prod_{v \in F}(p - v)$.

5.3 On the extended CWA

In classical two-valued logic, the extended CWA is a further generalization of the careful CWA which preserves consistency. However, it cannot be generalized to arbitrary multi-valued logics following our framework. In fact, if we now define $F(A,B,C)$ in Section 5.1 as the set of ground clauses such that $\varphi \in F(A,B,C)$, iff no predicate symbols occurring in φ belong to B, and there are no $\psi_1,\cdots,\psi_k \in A^+ \cup C^+ \cup C^-(k \geq 0)$ such that $S \vdash \psi_1 \cdots \psi_k \prod_{v\in T}(v - P\varphi)$ and $S \not\vdash \psi_1 \cdots \psi_k$, and let

$$NF = \{\prod_{v\in F}(P\varphi - v)|\varphi \in F(A,B,C)\}, ECWA(S;A,B,C) = PS \cup NF,$$

then $ECWA(S;A,B,C)$ may be inconsistent even if S is consistent. We take a look at the following example:

In Kleene's three-valued logic, let $S = \{\neg p(a), q(b)\}, A = \{p\}, B = \{q\}, C = \emptyset$. Then $PS|H = \{p(a), 1 - q(b)\}$, and it is not hard to examine that $\varphi_1 = p(a), \varphi_2 = p(b), \varphi_3 = \neg p(a) \wedge \neg p(b)$ and $\varphi_4 = \sim p(b) \wedge \sim \neg p(b)$ belong to $F(A,B,C)$, where $\sim$ is the weak negation we ever mentioned.
$\psi_1 = P\varphi_1(P\varphi_1 - 1/2) = p(a)(p(a) - 1/2)$,
$\psi_2 = P\varphi_2(P\varphi_2 - 1/2) = p(b)(p(b) - 1/2)$,
$\psi_3 = P\varphi_3(P\varphi_3 - 1/2) = 2(p(a) - 1)(p(a) - 1/2)(p(b) - 1)(p(b) - 1/2)$
$\psi_4 = P\varphi_4(P\varphi_4 - 1/2) = 2p(b)(1 - p(b))$
are in NF. But $(PS|H, \{\psi_1,\psi_2,\psi_3,\psi_4\}, AHB) = (1)$, $ECWA(S;\ A,\ B,\ C)$ is inconsistent.

Note that here ψ_3 and ψ_4 have been modulo AHB. Actually, for an m-valued logic, we only need to deal with the polynomial forms whose degrees are less than m.

6 Concluding remarks

In this paper, we discussed several formalizations of the CWA in multi-valued logics. From theoretical point of view, they applies in a uniform manner to any finitely multi-valued logics. From practical point of view, we adopted the language of polynomial forms instead of clauses. The related concepts can therefore be defined in an algebraic manner, and various computations can be done by operating polynomial ideals. Of course we can also use the resolution-based method[1] to operate the provability and consistency problems. However, this requires adding a new connective to the underlying logic. Our treatment seems easier to understand, and bears all the merits declared in [13].

So far we haven't yet implement our method. This is one of our future topics. There are also some other problems which need to be investigated further. One is how to decide whether or not there exists a clause corresponding to a given polynomial form. Since we wish to know the logic meaning of what we have derived, this problem is meaningful. Another is the descriptive power of clauses in various multi-valued logics. We know that it is enough to focus on such formulas in classical two-valued logic.

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