Symbolic Methods for the Equivalence Problem of Systems of Implicit Ordinary Differential Equations

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This contribution deals with the equivalence problems for systems of implicit ordinary differential equations of the following type

$$f^{i_e}\left(t, z, \dot{z}\right) = 0 \ , \quad i_e = 1, \ldots, n_e \tag{1}$$

with $z \in \mathbb{R}^q$, $q \leq n_e$. The time derivative of z is denoted by $\dot{z}$. Equivalence means that every solution of the original set of equations is a solution of some normal form and vice versa

$$\dot{x}^{i_e} = f^{i_e}\left(t, x, u\right) \ , \qquad \dot{x}^{i_e} = f^{i_e}\left(t, x, u^{(n)}\right) \ ,$$

where the variables x describe the *state* and u the *input* of the system. The symbol $u^{(n)}$ indicates that derivatives up to the order n of the input are admissible. The mathematical investigations are based on the theory of jet-bundles [1], the system (1) is identified with the submanifold in a suitable jet-space, defined by the equations. This approach allows us to combine methods from differential geometry and elimination theory.

The equivalence problem will be solved for the well-determined case, where the number of equations n_e and unknowns q is equal, as well as for the under-determined case with $q - n_e = 1$. For the case $q - n_e > 1$ several solutions will be presented, but we leave the problem open, how to minimize the number of inputs, whose derivatives appear in the normal form. Apart from the theoretical results we present several sketches for computer algebra based algorithms that are necessary to solve these problems efficiently.

[1] Saunders D.J.: The Geometry of Jet Bundles, London Mathematical Society Lecture Notes Series 142, Cambridge University Press, 1989.