Multivariate polynomials and Newton-Puiseux expansions

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In this paper, we will be interested in the local resolution of equations of the type:

$$f(y) = 0 \text{ with } f \in \mathbb{C}[x_1, \dots x_N, y]$$

To express the solutions, we will need a generalization for several variables of the Puiseux series: the series with exponents in a cone. To compute these series, we use an extension of the classical Newton polygon, the Newton polyhedron.
After a first part, in which we give some definitions and properties about elements of convex geometry and series with exponents in a cone, we present a resolution algorithm due to J. McDonald. We will take his work as starting point for the resolution process and improve it to obtain an algorithm computing what we called a full set of solutions. That is to say a set of couples composed of a cone and the associated series expansions of the solutions. More precisely, we consider the discriminant of f, with respect to the variable y, and the fan of its Newton polyhedron. To each cone σ of this fan, we can associate pathes of the Newton polyhedron of f and solutions expansions with exponents in a translate of σ.
For example, for the equation:

$$f(x_1, x_2, y) = x_2 y^2 + y^2 + x_2^2 y + x_1^2 y + x_1^2 x_2 y - x_1 x_2$$

we find three cones and for each cone we compute two solutions.