Database of graded rings and K3 surfaces in Magma

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K3 surfaces are one of the kinds of algebraic surface which naturally generalise elliptic curves. They are also very closely related to canonical models of curves and lie right in the heart of the classification of surfaces. Elliptic curves can be expressed in various standard formats: a plane cubic in Weierstrass form, an intersection of two quadrics in space, and so on. The standard first example of a K3 surface is defined by a homogeneous degree 4 polynomial in 4 variables (compare that with degree 3 in 3 variables for an elliptic curve). But there are very many other K3 surfaces in interesting formats.

I will report on a database of K3 surfaces in the computer algebra system Magma. This is built using Hilbert series methods which describe the degrees of variables and equations in some graded ring. The methods generate candidates for K3 surfaces which one must then show actually exist with the desired properties. The most interesting aspect of this is the way in which the inductive construction of the database helps to prove the existence of the objects which it contains.

I will show examples of K3 surfaces and give a description of the Hilbert series methods used to find candidates. Then I will indicate how the method of "unprojection" in conjunction with the building of the database can show the existence of these rather complicated graded rings. This is joint work with Selma Altınok and Miles Reid.