

# Linear Two-point Boundary-value Problem with Polynomial Coefficients

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We consider a defect control algorithm for computing an approximate solution to a linear, second-order, ordinary differential equation with polynomial coefficients. As the boundary conditions are given at two points, we have a two-point boundary-value problem. We express the second derivative as a translated Legendre polynomial with unknown coefficients and integrate twice to find an approximate solution. We substitute the polynomial into the differential equation to compute the defect and express this defect in an orthogonal polynomial basis, using translated Legendre polynomials. Then we equate the low-order terms to zero to generate a banded system of linear equations that we solve for the unknown coefficients. In some cases we are able to approximate the solution over the entire interval with a relative error less than  $10^{-100}$ .