F. Winkler 12 April 2005

Commutative Algebra & Algebraic Geometry
SS 2005

(4) Consider $I = \langle x_2x_1 - 2, x_3x_1 - 2\rangle$ in $\mathbb{Q}[x_1, x_2, x_3]$. Determine I_1 and $\pi_1(V(I))$ and compare them.

(5) Exhibit a maximal chain of irreducible algebraic sets in $\mathbb{A}(\mathbb{C})^3$.

(6) Prove the existence of polynomials f_i in Theorem 4.2.6.

(7) Consider the polynomial

$$f(x, y, z) = x^2z^2 + xyz^2 - 2xyz + 3.$$

Determine a linear change of coordinates L such that the leading coefficient of $L(f)$ w.r.t. z is a constant. Which linear transformations of the form used in the proof of Lemma 4.2.1 do NOT work?

(8) Let

$$f = xz + y - 1, \quad g = z^2 + xy + y.$$

Let

$$h_1 = yz - z - x^2y - xy, \quad h_2 = yz - x^2y - xy.$$

Determine whether h_1, h_2 are in the radical of the ideal (over $\mathbb{Q}$) generated by f^2 and g.