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Commutative Algebra & Algebraic Geometry
SS 2005

(14) Exercise 7.1.3 in Lecture Notes.

(15) Prove Euler's Formula for homogeneous polynomials $F(x_1, \ldots, x_n)$:

$$\sum_{i=1}^{n} x_i \cdot \frac{\partial F}{\partial x_i} \;=\; \deg(F) \cdot F.$$

(16) Consider the variety $\mathcal{V}$ in $A^3(C)$ defined by $x^2+y^2-4 = (x-2)^2+y^2+z^2-16 = 0$. Let $f = 4x + 15z^2$ and $g = 16y^2 + z^4 - 8$. Decide whether $f = g$ as polynomial functions on $\mathcal{V}$.

(17) Consider the circle $\mathcal{C}$ in $A^2(C)$ defined by $x^2 + y^2 = 1$ and the polynomial mapping $\varphi(x, y) = (x+y, x+2y, x+3y)$ from A^2 to A^3. What is the image of $\mathcal{C}$ under φ? Give defining equations for $\varphi(\mathcal{C})$. Is $\varphi(\mathcal{C})$ irreducible?

(18) Let $\mathcal{C}$ be the parabola defined in $A^2(C)$ by $y - 2x^2 = 0$. Determine the poles of the rational function $\varphi(x, y) = \frac{2x}{y-2x}$ on $\mathcal{P}$.