F. Winkler 24 May 2005

Commutative Algebra & Algebraic Geometry
SS 2005

(19) Consider the linear system $\mathcal{S}_3$ of curves of degree 3.

(a) Is it possible to determine, for any given 3 points P_1, P_2, P_3 in $\mathbb{P}^2(\mathbb{C})$, to find an element $\mathcal{C} \in \mathcal{S}_3$ having all these points as double points? If so, then determine such a curve for the points $(0:0:1), (0:1:1), (1:0:1)$.

(b) Is it possible to determine, for any given 4 points P_1, P_2, P_3, P_4 in $\mathbb{P}^2(\mathbb{C})$, to find an element $\mathcal{C} \in \mathcal{S}_3$ having all these points as double points? If so, then determine such a curve for the points $(0:0:1), (0:1:1), (1:0:1), (1:1:1)$.

(20) Compute the intersection multiplicities at the intersection points of the curves $\mathcal{C}$ and $\mathcal{D}$ defined by the polynomials

$$F(x,y,z) = x^3 + y^3 - 2xyz,$$
$$G(x,y,z) = 2x^3 - 4x^2y + 3xy^2 + y^3 - 2y^2z.$$

(21) Consider a rational curve $\mathcal{C}$ and a parametrization $\mathcal{P}$ of $\mathcal{C}$. Is it true that if the degree of $\mathcal{P}$ is prime then $\mathcal{P}$ is proper? If not, what are the exceptions.

(22) Prove that the curve

$$f(x,y) = y^4 + x - \frac{75}{8}\,x^2y^2 + \frac{125}{8}\,x^3y - \frac{1875}{256}\,x^4$$

is parametrizable by lines and compute a proper parametrization.