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Commutative Algebra & Algebraic Geometry
SS 2005

(23) Let $R(t) \in K(t)$ be non–constant. Prove that the following statements are equivalent:

(i) $R(t)$ is invertible.

(ii) $R(t)$ is linear.

(iii) $R(t) = \frac{at+b}{ct+d}$, where $a, b, c, d \in K$ and $ad - bc \neq 0$.

(24) Let $R(t) = \frac{p(t)}{q(t)} \in K(t)$ be a non–constant rational function in reduced form, let $U = \{\alpha \in K \mid q(\alpha) \neq 0\}$, and let $R : K \longrightarrow K$ be the rational mapping induced by $R(t)$. Prove that the cardinality of the set $K \setminus R(U)$ is less or equal 1.

(25) Consider the polynomial

$$f(x, y) = x^4 + x^2y^2 - xy^3 + 2y^4 - 2x^3 - x^2y - 4y^3 + x^2 + xy + 2y^2$$

defining an algebraic curve $\mathcal{C}$.

(a) What is the genus of the curve $\mathcal{C}$, and what are its singular points?

(b) Is $\mathcal{C}$ rationally parametrizable? If so, determine a parametrization.