# Unification Theory

Exercises

SS2008

For the definitions and notation see the course materials.

1. Prove:

   **Theorem 1 (Elementary Properties of Substitutions).**
   - *Composition of substitutions is associative.*
   - *For all $\mathcal{X} \subseteq \mathcal{V}$, $t$ and $\sigma$, if $vars(t) \subseteq \mathcal{X}$ then $t\sigma = t\sigma|_{\mathcal{X}}$.*
   - *For all $\sigma$, $\vartheta$, and $t$, if $t\sigma = t\vartheta$ then $t\sigma|_{vars(t)} = t\vartheta|_{vars(t)}$*

2. Prove:

   **Theorem 2.** *A substitution $\sigma$ is idempotent iff $dom(\sigma) \cap vran(\sigma) = \emptyset$.*

3. Prove:

   **Theorem 3.** *For any $\sigma$ and $\vartheta$, $\sigma \eqsim \vartheta$ iff there exists a variable renaming substitution $\eta$ such that $\sigma\eta = \vartheta$.*

4. Implement the Recursive Descent Algorithm in your favorite programming language.
5. Bring a counterexample that shows that removing the first line from the Recursive Descent Algorithm (**if** $s$ is a variable **then** $s := s\sigma$; $t := t\sigma$) destroys soundness.
6. Remove the first line from the Recursive Descent Algorithm (**if** $s$ is a variable **then** $s := s\sigma$; $t := t\sigma$), but modify the last line (**else** $\sigma := \sigma\{s \mapsto t\}$) in such a way that the obtained algorithm is correct.
7. Show the steps of the matching algorithm discussed in the class to find all substitutions that match the pattern term $f(x, f(a, x))$ to a subterm of $f(a, f(f(b, a), f(a, f(b, a))))$.
8. Slide 29 in 2008SS_04_E-Unification.pdf: Show that collection of all mgu's of $\Gamma'$s is a complete set of $C$-unifiers of $\Gamma$.
9. Do the exercise on the slide 63 in 2008SS_04_E-Unification_1.pdf.
10. Slide 65 in 2008SS_04_E-Unification_1.pdf: Show that the set of solutions is complete.
11. Slide 20 in 2008SS_05_E-Unification_2.pdf: Prove the theorem.
12. Find a counterexample that shows that if the set of rewrite rules $R$ is not confluent, then basic narrowing with $R$ is incomplete.