Algebraic and Discrete Methods in Biology

Propositional Logic.
Logical Equivalence and Normal Forms

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Outline

Logical Equivalence

Normal Forms

Logical Equivalence

We say *"φ is equivalent to ψ"*, (also denoted as $\varphi \equiv \psi$) if and only if $\varphi \models \psi$ and $\psi \models \varphi$.

Equivalent formulae have exactly the same semantics, that is, their semantic functions are equal for every interpretation.

Equivalences are important because they allow us to transform individual formulae, without changing the truth. Even parts of formulae can be transformed.

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Important Equivalences

Commutativity

$$\varphi \wedge \psi \equiv \psi \wedge \varphi$$
$$\varphi \vee \psi \equiv \psi \vee \varphi$$

Associativity

$$\varphi_1 \wedge (\varphi_2 \wedge \varphi_3) \equiv (\varphi_1 \wedge \varphi_2) \wedge \varphi_3$$
$$\varphi_1 \vee (\varphi_2 \vee \varphi_3) \equiv (\varphi_1 \vee \varphi_2) \vee \varphi_3$$

Idempotence

$$\varphi \wedge \varphi \equiv \varphi$$
$$\varphi \vee \varphi \equiv \varphi$$

Example

The following equivalence can be obtained using the above rules:
$(A \vee ((A \vee B) \vee (C \vee B))) \vee C \equiv (A \vee B \vee C)$

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Properties of negation

$$\neg\neg\varphi \equiv \varphi$$

$$\neg\varphi \lor \varphi \equiv \mathbb{T}$$

$$\neg\varphi \land \varphi \equiv \mathbb{F}$$

$$\neg(\varphi \lor \psi) \equiv (\neg\varphi) \land (\neg\psi)$$

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Elimination of $\Rightarrow$ and $\Leftrightarrow$

$$\varphi \Rightarrow \psi \;\equiv\; (\neg\varphi) \vee \psi$$

$$\begin{aligned}\varphi \Leftrightarrow \psi \;&\equiv\; (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi)\\ &\equiv\; (\neg\varphi \vee \psi) \wedge (\neg\psi \vee \varphi)\\ &\equiv\; (\varphi \wedge \psi) \vee (\neg\varphi \wedge \neg\psi)\end{aligned}$$

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Important Equivalences

Properties of the truth constants $\mathbb{T}$ and $\mathbb{F}$

$$\mathbb{T} \vee \varphi \equiv \mathbb{T}$$

$$\mathbb{T} \wedge \varphi \equiv \varphi$$

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$\mathbb{T} \Rightarrow \varphi \;\equiv\; \varphi$

because $\mathbb{T} \Rightarrow \varphi \;\equiv\; (\neg\mathbb{T}) \vee \varphi \;\equiv\; \mathbb{F} \vee \varphi \;\equiv\; \varphi$

$\mathbb{F} \Rightarrow \varphi \;\equiv\; \mathbb{T}$ (false implies anything)

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Summary: Laws for transformation on formulae

$$
\begin{aligned}
\varphi \Leftrightarrow \psi &\equiv (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi) \\
\varphi \Rightarrow \psi &\equiv \neg\varphi \vee \psi \\
\varphi \vee \psi &\equiv \psi \vee \varphi \\
\varphi \wedge \psi &\equiv \psi \wedge \varphi \\
(\varphi \vee \psi) \vee \chi &\equiv \varphi \vee (\psi \vee \chi) \\
(\varphi \wedge \psi) \wedge \psi &\equiv \varphi \wedge (\psi \wedge \chi) \\
(\varphi \vee \psi) \wedge \chi &\equiv (\varphi \wedge \chi) \vee (\psi \wedge \chi) \\
(\varphi \wedge \psi) \vee \chi &\equiv (\varphi \vee \chi) \wedge (\psi \vee \chi) \\
\psi \wedge \mathbb{F} &\equiv \mathbb{F} \\
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\psi \wedge \neg\psi &\equiv \mathbb{F} \\
\psi \vee \neg\psi &\equiv \mathbb{T} \\
\neg\neg\psi &\equiv \psi \\
\neg(\varphi \vee \psi) &\equiv \neg\varphi \wedge \neg\psi \\
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\end{aligned}
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Definition
A *literal* is an atom or the negation of an atom.
In the propositional calculus the atoms are the logical variables.

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A formula φ is in a *conjunctive normal form* iff $\varphi \doteq A_1 \wedge \cdots \wedge A_n$, for some $n \geq 1$, where each of $A_1, \ldots, A_n$ is a disjunction of literals.

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Normal Forms

Example

$\varphi_1 \ \hat{=}\ (P \lor \neg Q \lor R) \land (\neg P \lor Q)$

Example

$\varphi_2 \ \hat{=}\ (P \land R) \lor (\neg Q \land R) \lor (P \land Q \land R)$

Example

$\varphi_3 \ \hat{=}\ (P \land R) \lor (Q \land R) \lor (P \land \neg\neg Q)$

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$$\varphi_2 \doteq (P \wedge R) \vee (\neg Q \wedge R) \vee (P \wedge Q \wedge R)$$

Example

$$\varphi_3 \doteq (P \wedge R) \vee (Q \wedge R) \vee (P \wedge \neg\neg Q)$$

Transformation into Normal Forms

Step 1 **Use the laws:**

$$\varphi \Leftrightarrow \psi \;=\; (\varphi \Rightarrow \psi) \land (\psi \Rightarrow \varphi)$$
$$\varphi \Rightarrow \psi \;=\; \neg\varphi \lor \psi$$

Step 2 **Use the laws:**

$$\neg(\neg\psi) \;=\; \psi$$
$$\neg(\varphi \lor \psi) \;=\; \neg\varphi \land \neg\psi$$
$$\neg(\varphi \land \psi) \;=\; \neg\varphi \lor \neg\psi$$

Step 3 **Use the laws:**

$$(\varphi \lor \psi) \land \chi \;=\; (\varphi \land \psi) \lor (\psi \land \chi)$$
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Obtain *dnf* for the formula $\varphi_1 \ \dot{=}\ (P \vee \neg Q) \Rightarrow R$

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Obtain *cnf* for the formula $\varphi_1 \ \dot{=}\ (P \vee \neg Q) \Rightarrow R$

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Obtain *dnf* for the formula $\varphi_2 \ \dot{=}\ (P \wedge (Q \Rightarrow R)) \Rightarrow S$

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