Algebraic and Discrete Methods in Biology

Proving in Natural Style

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Proving in Natural Style

Proving in natural style means proving in a style which is similar to the style which scientists naturally use when they do proofs.

It is important to understand very well the principles of proving in natural style, because this allows us to construct proofs in an intuitive way, and also helps us in understanding the proofs done by other people or by computer programs.

We have seen several equivalences, which allow transformations of one formula into another. Here we look situations of logical consequences, which allow us to infer new formulae from one or more given formulae.

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Proving in Natural Style

φ	$\models$	φ	Trivial
$\varphi \wedge \psi$	$\models$	φ	True conjunct
$\varphi,\ \psi$	$\models$	$\varphi \wedge \psi$	Conjunction
φ	$\models$	$\varphi \vee \psi$	Disjunction
$\varphi,\ \varphi \Rightarrow \psi$	$\models$	ψ	Modus ponens
$\neg\psi,\ \varphi \Rightarrow \psi$	$\models$	$\neg\varphi$	Modus tollens
$\neg\varphi,\ \varphi \vee \psi$	$\models$	ψ	Resolution

Proving in Natural Style

Example

Using the natural style proving, show that $A \vee B$ is a logical consequence of $P \wedge Q,\ Q \Rightarrow C,\ B \Rightarrow \neg P,\ (C \wedge \neg B) \Rightarrow A$.

Example

Using the natural style proving, show that $((A \vee B) \Rightarrow C) \models ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

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Example

Using the natural style proving, show that
$((A \vee B) \Rightarrow C) \models ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

Useful proof rules in propositional logic

Goal oriented rules

When the goal is $\neg\varphi$:
Assume φ and try to obtain a contradiction.

When the goal is $\varphi \wedge \psi$:
Prove φ and prove ψ.

When the goal is $\varphi \vee \psi$:
Assume $\neg\varphi$ and prove ψ.

When the goal is $\varphi \Rightarrow \psi$:
Assume φ and prove ψ (*"the deduction rule"*).

When the goal is $\varphi \Leftrightarrow \psi$:
Assume φ and prove ψ, and then assume ψ and prove φ.

Useful proof rules in propositional logic

Assumption oriented rules

When an assumption is $\neg(\neg\varphi)$:
Transform it into φ.

When an assumption is $\neg(\varphi \wedge \psi)$:
Transform it into $(\neg\varphi) \vee (\neg\psi)$.

When an assumption is $\neg(\varphi \vee \psi)$:
Transform it into two assumptions $\neg\varphi$ and $\neg\psi$.

When an assumption is $\neg(\varphi \Rightarrow \psi)$:
Transform it into two assumptions φ and $\neg\psi$.

When an assumption is $\neg(\varphi \Leftrightarrow \psi)$:
Transform it into two assumptions $\varphi \vee \psi$ and $(\neg\varphi) \vee (\neg\psi)$.

Useful proof rules in propositional logic

Assumption oriented rules

When an assumption is $\varphi \wedge \psi$:
Transform it into two assumptions φ and ψ.

When an assumption is $\varphi \vee \psi$:
Split the proof into two branches, on one assume φ and on the other one assume ψ (*"proof by cases"*).

When an assumption is $\varphi \Rightarrow \psi$:
If φ is already known (or it can be proven), obtain ψ (*"modus ponens"*).

When an assumption is $\varphi \Leftrightarrow \psi$:
Use any one of the implications, depending on the other formulae which are known.

Proving in Natural Style

Example

Show that $(P \Rightarrow Q) \ \wedge \ (Q \Rightarrow R) \ \models \ (P \Rightarrow R)$.

Example

Show that $\models \ (A \wedge (A \Rightarrow B)) \Rightarrow B$.

Example

Show that $C \ \models \ C \ \wedge \ ((A \wedge (A \Rightarrow B)) \Rightarrow B)$.

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Show that $(A \vee B) \;\Rightarrow\; C \models (A \Rightarrow C) \;\wedge\; (B \Rightarrow C)$.

Example

Show that $(A \Rightarrow C) \;\wedge\; (B \Rightarrow C) \models (A \vee B) \;\Rightarrow\; C$.

Example

Show that $(A \wedge B) \;\Rightarrow\; C \models (A \Rightarrow C) \;\vee\; (B \Rightarrow C)$.

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Show that $(A \Rightarrow C) \;\vee\; (B \Rightarrow C) \models (A \wedge B) \;\Rightarrow\; C$.

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