CLP

Logic Programming

versus

Constraint Logic Programming

Introduction to

CLP systems

Unification

Equality of Two Terms

PROLOG

Syntactical Unification

CLP

Syntactical Unification

and

Unification over

Computational Domains

Example

PROLOG

```
X + 5
```

a binary operation, +,
with two arguments, X and 5

Plus has no semantical meaning other than being a binary operation

There is no information about integer domains

Syntactic Unification

```
| ?- X+2=Y+2.
Y = X ?


| ?- X+2=2+Y.
X = 2,
Y = 2 ?


| ?- X+7=9+Y.
X = 9,
Y = 7 ?

| ?- 1+1=1+1.
yes
```

No Computation

```
| ?- 1+1=1+1.
yes

| ?- 2=2.
yes

| ?- 1+1=2.

no

| ?- X+7=9+Y.
X = 9,
Y = 7 ?

yes
| ?- X+7=Y+9.
no

| ?- X=5,X>3.
X = 5 ?
```

```
| ?- X>2.
INSTANTIATION ERROR: in expression

| ?- N1=6,N is N1-1.
N = 5,
N1 = 6 ?

| ?- N is N1-1.
INSTANTIATION ERROR: in expression
```

Fibonacci Sequence

```
fib(0,1).
fib(1,1).
fib(N,R):- N1 is N-1,
           fib(N1,R1),
           N2 is N-2,
           fib(N2,R2),
           R is R1+R2.
```

Functional Formulation

```
| ?- fib(0,X).

X = 1 ?

yes
| ?- fib(1,X).

X = 1 ?

yes
| ?- fib(2,X).

X = 2 ?

yes
| ?- fib(3,X).

X = 3 ?

yes
```

```
| ?- fib(X,3).
INSTANTIATION ERROR: in expression
```

fib(3,X)

```
| ?- fib(3,X).
 + 1  1  Call: fib(3,_75) ?
 + 2  2  Call: _360 is 3-1 ?
 + 2  2  Exit: 2 is 3-1 ?
 + 3  2  Call: fib(2,_352) ?
 + 4  3  Call: _988 is 2-1 ?
 + 4  3  Exit: 1 is 2-1 ?
 + 5  3  Call: fib(1,_980) ?
 + 5  3  Exit: fib(1,1) ?
 + 6  3  Call: _973 is 2-2 ?
 + 6  3  Exit: 0 is 2-2 ?
 + 7  3  Call: fib(0,_965) ?
 + 7  3  Exit: fib(0,1) ?
 + 8  3  Call: _352 is 1+1 ?
 + 8  3  Exit: 2 is 1+1 ?
 + 3  2  Exit: fib(2,2) ?
 + 9  2  Call: _345 is 3-2 ?
 + 9  2  Exit: 1 is 3-2 ?
 + 10  2  Call: fib(1,_337) ?
 + 10  2  Exit: fib(1,1) ?
```

```
+ 11  2  Call: _75 is 2+1 ?
+ 11  2  Exit: 3 is 2+1 ?
+ 1  1  Exit: fib(3,3) ?
```

fib(X,3)

```
| ?- fib(X,3).
 + 1  1  Call: fib(_61,3) ?
 + 2  2  Call: _360 is _61-1 ?
 + 2  2  Exception: _360 is _61-1 ?
 + 1  1  Exception: fib(_61,3) ?
INSTANTIATION ERROR: in expression
```

Integer as Term

```
ned(0,s(0)).
ned(N,s(S)) :- N1 is N-1, ned(N1,S).
```

Integers as Terms

```
peano(0,s(0)).
peano(N,s(S)) :- N1 is N-1, peano(N1,S).

| ?- peano(0,S).
S = s(0) ?

yes
| ?- peano(1,S).
S = s(s(0)) ?

yes
| ?- peano(5,S).
S = s(s(s(s(s(s(0)))))) ?
```

What would addition look like?

Generate Integers

```
nnnn(0).
nnnn(N) :- nnnn(N1), N is N1 + 1.


| ?- nnnn(0).
yes
| ?- nnnn(1000).
yes
| ?- nnnn(10).
yes

| ?- nnnn(X).
X = 0 ? ;
X = 1 ? ;
X = 2 ? ;
X = 3 ? ;
X = 4 ? ;
X = 5 ? ;
```

Generate and Test

Depth First Search

Generate a Possibility

Test to see if true

Inefficient

Does not use the power of the theory of (for example) natural numbers

Wish

To be able to work within other domains

CLP

CLP allows
Semantic Interpretation

Under Domains
Integers
Rationals

Core Idea
Replace the
Computational Heart
of PROLOG
with a
Constraint Domain

Reals: CLP(R)
Integers: CLP(Z)
Rationals: CLP(Q)

Arithmetical Interpretation in these domains

Constraint

A constraint is a limitation
of the full possible range of values

The variable `X`
over the integers
Can have any value

```
X < 2
```

puts a *constraint* on the values of `X`
Now it can only take values less than 2.

```
X < 2
```

Makes a statement
transfering the objects `X` and `2`
from the integer domain
to the logical domain
(that of Logic Programming)

Formulas

Different statements make constraints on the values that the variables involved may take within the domain

Domain

Why are domains important?

The interpretation depends on the
Algebraic Domains

Examples

Does a solution exist
A yes/no question

$X^2 - 4$ has a solution over
Integers, Reals and Complex

$$X^2 - 3$$

has no solutions over the integers
but over the reals and complex

$$X^2 + 4$$

has no solutions over
the integers or reals
but does have a solution over the complex
numbers

Some Constraints

```
1 ?- X*X+Y*Y > 0.
0 < X
real(Y)

*** Retry? ;
0 < Y
real(X)

*** Retry? ;
Y < 0
real(X)

*** Retry? ;
X < 0
real(Y)

*** Yes
2 ?- X*X+Y*Y < 0.
*** No
```

Some Constraints

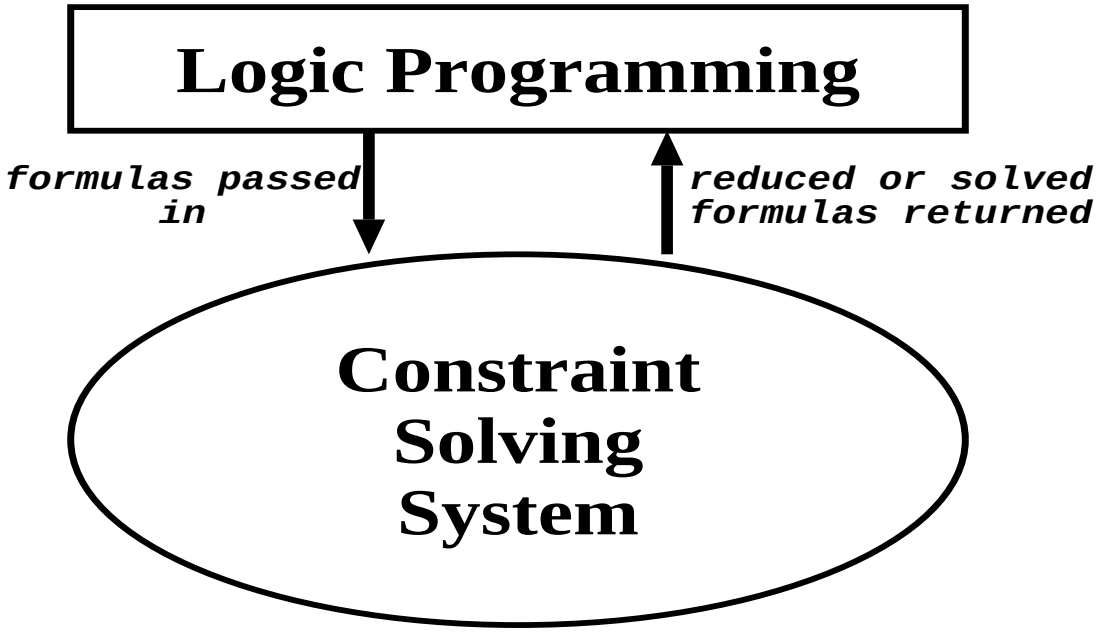
```
4 ?- 9*X*X - 4*Y*Y + 8*Y = 4.
9*X*X - 4*Y*Y + 8*Y < 4.
X = -0.666667*Y + 0.666667


5 ?- 9*X*X - 4*Y*Y + 8*Y = 4.
9*X*X - 4*Y*Y + 8*Y < 4.
X = -0.666667*Y + 0.666667


6 ?- X*X + (Y-3)*(Y-3) + (Z+5)*(Z+5) <= 0.
Z = -5
Y = 3
X = 0
*** Yes

7 ?- X*Y >= 1.

X*Y - 1 >= 0
real(Y)
```

real(X)

Implementation Problems

```
?- X=5*Y,X*X+Y*Y=25.

Y = 0.980581
X = 4.9029

*** Retry? ;

Y = -0.980581
X = -4.9029



?- X*X+Y*Y=25,X=5*Y.

X = 5*Y
X*X + Y*Y - 25 = 0
```

Architecture



Fibonacci Revisited

```
fib(0,1).
fib(1,1).
fib(N,R1+R2) :- N >= 2,
fib(N-1,R1),
fib(N-2,R2).

11 ?- fib(0,X).
X = 1


?- fib(10,X).
X = 89


?- fib(X,89).
X = 10


?- fib(X,100).
Stacksize = 100000
```

```
Fatal Error: Stack overflow
```

```
%%% The Famous Balanced Meal .

lightMeal(H,M,D) :-
   horsDoeuvre(H,I),
   mainCourse(M,J),
   dessert(D,K)
   I>=0, J>=0, K>=0, I+J+K=<10.

mainCourse(M,I) :- meat(M,I).
mainCourse(M,I) :- fish(M,I).

horsDoeuvre(radishes,1).
horsDoeuvre(pate,6).

meat(beef,5).
meat(pork,7).

fish(sole,2).
fish(tuna,4).

dessert(fruit,2).
```

```
dessert(ice_cream,6).
```