to be prepared for Januar 9

Exercise 42. Use resultants to find the implicit representation, i.e. a polynomial equation just in x, y, and z of the parametrized surface

$$\begin{aligned} x &= 1 + s + t + st \\ y &= 2 + s + st + t^2 \\ z &= s + t + s^2 \end{aligned}$$

Exercise 43. Let I be a unique factorization domain, $f, g \in I[x]$ with $\deg(f) > 0, \deg(g) > 0$. Show that there are polynomials $a, b \in I[x]\backslash 0$ such that $\mathrm{res}(f, g) = af + bg$.

Exercise 44. Let $f, g \in k[x]$ be polynomials whose roots are $\zeta_1, \ldots, \zeta_m$ and $\eta_1, \ldots, \eta_n$ respectively. Prove that

$$\mathrm{res}_x(f, g) = LC(f)^{\deg(g)} LC(g)^{\deg(f)} \prod_{i=1}^{m} \prod_{j=1}^{n} (\zeta_i - \eta_j)$$

Exercise 45. Compute the factorization in $\mathrm{GF}(27)[x]$ of the squarefree polynomial

$$x^{10}+(1+\alpha^2+\alpha)x^9+(2\alpha+\alpha^2+1)x^7+(2\alpha^2+2)x^6+(\alpha+\alpha^2)x^4+(2+2\alpha+\alpha^2)x^3+(1+\alpha^2+\alpha)x^2+(1+\alpha)x+1+\alpha$$

into irreducible factors, where α is a root of the polynomial $1 - x + x^3 \in \mathbb{Z}_3[x]$.

Exercise 46. Compute the factorization of the polynomial

$$\begin{aligned} & x^{19} + 14x^{18} + 23x^{17} + 3x^{16} + 22x^{15} + 25x^{14} + 27x^{13} + 22x^{12} + 58x^{11} + 38x^{10} + \\ & + 42x^9 + 74x^8 + 55x^7 + 68x^6 + 62x^5 + 38x^4 + 39x^3 + 18x^2 + 7x + 4 \end{aligned}$$

into irreducibles in $\mathbb{Z}[x]$ by the modular method.