to be prepared for Januar 23

Exercise 52. Generalize the notion of norm to multivariate polynomials. In analogy to chapter 5.4, describe the basic notion of a multivariate norm.

Exercise 53. According to Theorem 4.3.3 a solution of $\mathrm{res}_{x_r}(a, b) = 0$ can be extended to a common zero of a and b, in case the leading coefficient of a or b is a nonzero constant. This can be achieved by a suitable change of coordinates. Work out an algorithm for solving systems of algebraic equations by resultants along these lines.

Exercise 54.

1. Explain why the Euclidean algorithm can be considered as a special case of Gröbner base algorithm.

2. Considering a system of linear equations in indeterminates $x_1, \ldots, x_n$, discuss the relation between Gauss elimination and Gröbner bases.

Exercise 55. Study the literature (e.g. on Gröbner bases) and try to find an algorithmic answer to the following problem: Given two ideals $I, J \subseteq k[x_1, \ldots, x_n]$ in terms of generators $I = \langle f_1, \ldots, f_r \rangle$, $J = \langle g_1, \ldots, g_s \rangle$, find generators of the intersection $I \cap J$.

Exercise 56. Prove the following theorem.
Let $F \subseteq k[x_1, \ldots, x_n]$. The ideal congruence modulo $\langle F \rangle$ equals the reflexive-transitive-symmetric closure of the reduction relation $\longrightarrow_F$, i.e., $\equiv_{\langle F \rangle} = \longleftrightarrow^{\star}_F$.