to be prepared for 17th October

Exercise 6. In your favorite computer algebra system (Maple, Mathematica, Derive, ...) find out about possibilities for solving systems of polynomial equations.

1. Consider the system of equations

$$\begin{aligned} 2x^4 - 3x^2y + y^4 - 2y^3 + y^2 &= 0 \\ 4x^3 - 3xy &= 0 \\ 4y^3 - 3x^2 - 6y^2 + 2 &= 0. \end{aligned}$$

   Compute all solutions.

2. The same for

$$\begin{aligned} 1 + 8xy + 2y^2 + 8xy^3 + y^4 - 16x^2 &= 0 \\ 8x + 4y + 24xy^2 + 4y^3 &= 0 \\ 8y + 8y^3 - 32x &= 0. \end{aligned}$$

Exercise 7. Consider multiplication of (positive) integers in decimal notation. What is the number of operations on individual digits (time complexity) necessary for multiplying two integers of length m and n using

1. the classical algorithm?
2. the Karatsuba method ?

Exercise 8. The idea of Karatsuba can also be applied to the multiplication of polynomials. Determine the number of multiplications necessary for multiplying the polynomials

$$a_0x^3 + a_1x^2 + a_2x + a_3 \text{ und } b_1x^2 + b_2x + b_3$$

both with the classical and with the Karatsuba method.

Exercise 9. Consider the integers $a = 215712$, $b = 739914$. Determine the gcd of a and b and s, t such that $\gcd(a, b) = s * a + t * b$.

Exercise 10. An integral domain is a commutative ring $D \neq \{0\}$ without zero divisors, that means, $rs = 0 \Rightarrow r = 0 \lor s = 0$ $(\forall r, s \in D)$. Give a proof for the following statement.

1. If D is an integral domain, then also the polynomial ring $D[x]$.
2. Derive from this that - for arbitrary fields k - the ring $k[x_1, \ldots, x_n]$ is an integral domain.
3. Give similar arguments for the ring $D[[x]]$ of formal power series.