to be prepared for 24th October

Exercise 11. Compute the pseudo-quotient $q(x)$ and the pseudo-remainder $r(x)$ for the two integral polynomials

$$\begin{aligned} u(x) &= x^6 + x^5 - x^4 + 2x^3 + 3x^2 - x + 2 \quad \text{and} \\ v(x) &= 2x^3 + 2x^2 - x + 3. \end{aligned}$$

Exercise 12. Let R be an integral domain, $a, b \in R[x]$, $b \neq 0$, $m = \deg(a) \geq n = \deg(b)$, $\alpha, \beta \in R$. Prove:

$$\begin{aligned} \mathrm{pquot}(\alpha a, \beta b) &= \beta^{m-n}\alpha \cdot \mathrm{pquot}(a, b) \quad \text{and} \\ \mathrm{prem}(\alpha a, \beta b) &= \beta^{m-n+1}\alpha \cdot \mathrm{prem}(a, b). \end{aligned}$$

Exercise 13. Choose two different irreducible polynomials for constructing the Galois field $GF(2^3)$ in two essentially different ways. Demonstrate that your resulting fields are actually isomorphic.

Exercise 14. Let R be a ring of prime characteristic p and $a, b \in R$. Prove:

$$\begin{aligned} (a+b)^p &= a^p + b^p \\ (a+b)^{p^n} &= a^{p^n} + b^{p^n}, \text{ for } n \in \mathbb{N}. \end{aligned}$$

Exercise 15. Prove the following statement. *If k is a finite field then k is a simple extension of some $\mathbb{Z}_p$.*