to be prepared for 31th October

Exercise 16. Prove the following statement (Theorem 2.2.2)
Let R be an integral domain, $a(x), b(x) \in R[x], b \neq 0$, and $m = \deg(a) \geq n = \deg(b)$. There are uniquely defined polynomials $q(x), r(x) \in R[x]$ such that

$$\begin{array}{l} \mathrm{lc}(b)^{m-n+1} \cdot a(x) = q(x) \cdot b(x) + r(x) \quad and \\ r(x) = 0 \; or \; \deg(r) < \deg(b). \end{array}$$

Exercise 17. Let $\mathbb{Z}_5(\alpha)$ be the algebraic extension of $\mathbb{Z}_5$ by a root α of the irreducible polynomial x^5+4x+1. Compute the normal representation of $(ab)/c$, where $a = \alpha^3 + \alpha + 2$, $b = 3\alpha^4 + 2\alpha^2 + 4\alpha$, $c = 2\alpha^4 + \alpha^3 + 2\alpha + 1$.

Exercise 18. Let R be commutative ring, $R[x]$ the corresponding polynomial ring. Proof that $R[x]$ is a Euclidean domain if and only if R is a field.

Exercise 19. Solve the Chinese remainder problem

$$\begin{array}{rcl} r \equiv & 62 & \mod 79 \\ r \equiv & 66 & \mod 83 \\ r \equiv & 72 & \mod 89 \end{array}$$

over the integers both by the Lagrange and by the Newton method.

Exercise 20. Solve the Chinese remainder problem

$$\begin{array}{rcl} r(x) \equiv & 7x-2 & \mod x^2+x-1 \\ r(x) \equiv & 11x+8 & \mod x^2-x-1 \\ r(x) \equiv & -3x & \mod x^2+x+1 \end{array}$$

over $\mathbb{Q}[x]$ by the Lagrange and by the Newton method.