to be prepared for November 21st

Exercise 26. Consider the polynomials

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2 \ . \end{aligned}$$

Compute the gcd of f and g by the modular algorithm.

Exercise 27. Consider the bivariate polynomials

$$\begin{aligned} f(x,y) &= x^2y^3 - 5xy^3 + 6y^3 - 6xy^2 + 18y^2 + 2xy - 4y - 12, \\ g(x,y) &= x^2y^3 + 3xy^3 - 10y^3 - 6xy^2 - 30y^2 - 4xy + 8y + 24. \end{aligned}$$

Compute the gcd of f and g by the modular algorithm. Take care of leading coefficients.

Exercise 28. Compute the squarefree factorization of

1. $f(x) = x^6 - x^5 + x^3 - x^2$ over the field $\mathbb{Z}_3$.
2. $g(x) = x^7 + x^5 + x^4 + x^3 + x^2 + 1$ over $GF(9)$.

Exercise 29. Use a computer algebra system to find the square-free part of the polynomial

$$x^{11} - x^{10} + 2x^8 - 4x^7 + 3x^5 - 3x^4 + x^3 + 3x^2 - x - 1.$$

1. over $\mathbb{Z}$;
2. over $\mathbb{Z}_3$.

Exercise 30. Prove the following theorem [Theorem 4.4.2]. Let K be a field of characteristic 0, and $a(x_1, \ldots, x_n) \in K[x_1, \ldots x_n]$. Then a is squarefree if and only if $\gcd\left(a, \frac{\partial a}{\partial x_1}, \ldots, \frac{\partial a}{\partial x_n}\right) = 1$.