

to be prepared for November 29

**Exercise 31.** Let  $I$  be a unique factorization domain, and  $f, g \in I[x]$ . Write  $f \sim g$  if there is a unit  $\varepsilon$  with  $g = \varepsilon f$ . Prove the following:

1.  $\text{cont}(fg) \sim \text{cont}(f) \cdot \text{cont}(g)$
2.  $\text{pp}(fg) \sim \text{pp}(f) \cdot \text{pp}(g)$

**Exercise 32.** Let  $a(x)$  be a primitive polynomial with integer coefficients, and assume that  $a(x)$  is squarefree. Prove that there are only finitely many primes  $p$  such that  $a(x)$  is not squarefree modulo  $p$ .

**Exercise 33.** Try to derive an algorithm for producing the squarefree decomposition in the field  $\mathbb{Z}_p[x]$ ,  $p$  a prime.

**Exercise 34.** After producing squarefree factors apply Berlekamp's algorithm for computing the complete factorization in  $\mathbb{Z}_5$  of

$$x^7 + 4x^6 + 2x^5 + 4x^3 + 3x^2 + 4x + 2.$$

**Exercise 35.** Give an example of a polynomial  $f \in k[x]$  that is irreducible with multiple roots.