to be prepared for 20th November

Exercise 25. For $m \in \mathbb{Z}$ let $\mathbb{Z}_m$ denote the group $\mathbb{Z}/m\mathbb{Z}$. Prove the following statement:

If $k, n \in \mathbb{Z}$ are relatively prime then $\mathbb{Z}_{kn} \cong \mathbb{Z}_k \oplus \mathbb{Z}_n$.

Exercise 26. Prove the following statement. *Two nonzero polynomials $f(x)$ and $g(x)$ with coefficients in a unique factorization domain are similar iff they have equal primitive part.*

Exercise 27. Given the polynomials

$$\begin{aligned} f(x) &= x^7 - 3x^5 - 2x^4 + 13x^3 - 15x^2 + 7x - 1, \\ g(x) &= x^6 - 9x^5 + 18x^4 - 13x^3 + 2x^2 + 2x - 1 \end{aligned}$$

compute their gcd $h \in \mathbb{Z}[x]$. Check whether the integer factors of the resultant of f/h and g/h are unlucky primes in the modular approach to gcd computation.

Exercise 28. Prove the following theorem: *If I is a unique factorization domain, then so is $I[x]$.*

Exercise 29. Prove that every Euclidean domain is a principal ideal domain, and that every principal ideal domain is a unique factorization domain.