

*Logic Programming*  
*Using Data Structures*  
*Part 2*

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# Contents

Recursive Comparison

Joining Structures Together

Accumulators

Difference Structures

# Comparing Structures

Structure comparison:

- ▶ More complicated than the simple integers
- ▶ Have to compare all the individual components
- ▶ Break down components recursively.

# Comparing Structures. `ales`

## Example

`ales(X, Y)` **succeeds** if

- ▶ `X` and `Y` stand for atoms and
- ▶ `X` is alphabetically less than `Y`.

`ales(avocado, clergyman)` **succeeds**.

`ales(windmill, motorcar)` **fails**.

`ales(picture, picture)` **fails**.

## Comparing Structures. `aless`

**Success** First word ends before second:

```
aless (book, bookbinder) .
```

**Success** A character in the first is alphabetically less than one in the second:

```
aless (avocado, clergyman) .
```

**Recursion** The first character is the same in both. Then have to check the rest:

```
For aless (lazy, leather) check
```

```
aless (azy, eather) .
```

**Failure** Reach the end of both words at the same time:

```
aless (apple, apple) .
```

**Failure** Run out of characters for the second word:

```
aless (alphabetic, alp) .
```

# Representation

- ▶ Transform atoms into a recursive structure.
- ▶ List of integers (ASCII codes).
- ▶ Use built-in predicate `name`:

```
?- name(alp, [97,108,112]).  
yes
```

```
?- name(alp, X).  
X = [97,108,112] ?  
yes
```

```
?- name(X, [97,108,112]).  
X = alp ?  
yes
```

# First Task

**Convert** atoms to lists:

```
name(X, XL).  
name(Y, YL).
```

**Compare** the lists:

```
allessx(XL, YL).
```

**Putting together:**

```
alless(X, Y) :-  
    name(X, XL),  
    name(Y, YL),  
    allessx(XL, YL).
```

## Second Task

Compose `allessx`.

**Success** First word ends before second:

```
allessx([], [_|_]).
```

**Success** A character in the first is alphabetically less than one in the second:

```
allessx([X|_], [Y|_]) :- X < Y.
```

**Recursion** The first character is the same in both. Then have to check the rest:

```
allessx([H|X], [H|Y]) :- allessx(X, Y).
```

What about failing cases?



# Program

```
alless(X,Y):-  
    name(X,XL),  
    name(Y,YL),  
    alessx(XL,YL).
```

```
allessx([],[_|_]).  
allessx([X|_],[Y|_]):-X<Y.  
allessx([H|X],[H|Y]):-allessx(X,Y).
```

# Appending Two Lists

For any lists `List1`, `List2`, and `List3`

`List2` **appended** to `List1` is `List3` iff either

- ▶ `List1` is the empty list and `List3` is `List2`, or
- ▶ `List1` is a nonempty list and
  - ▶ the head of `List3` is the head of `List1` and
  - ▶ the tail of `List3` is `List2` **appended** to the tail of `List1`.

Program:

```
append([], L, L) .
```

```
append([X|L1], L2, [X|L3]) :- append(L1, L2, L3) .
```

# Using append

**Test** ?- append([a,b,c],[2,1],[a,b,c,2,1]).

**Total List** ?- append([a,b,c],[2,1],X).

**Isolate** ?- append(X,[2,1],[a,b,c,2,1]).

?- append([a,b,c],X,[a,b,c,2,1]).

**Split** ?- append(X,Y,[a,b,c,2,1]).

# Inventory Example

## Bicycle factory

- ▶ To build a bicycle we need to know which parts to draw from the supplies.
- ▶ Each part of a bicycle may have subparts.
- ▶ Task: Construct a tree-based database that will enable users to ask questions about which parts are required to build a part of bicycle.

# Parts of a Bicycle

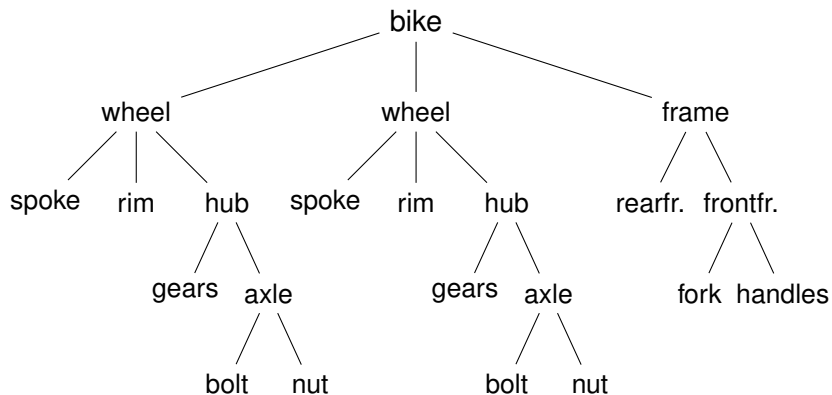
- ▶ **Basic parts:**

```
basicpart(rim).          basicpart(gears).  
basicpart(spoke).       basicpart(bolt).  
basicpart(rearframe).   basicpart(nut).  
basicpart(handles).     basicpart(fork).
```

- ▶ **Assemblies, consisting of a quantity of basic parts or other assemblies:**

```
assembly(bike, [wheel, wheel, frame]).  
assembly(wheel, [spoke, rim, hub]).  
assembly(frame, [rearframe, frontframe]).  
assembly(hub, [gears, axle]).  
assembly(axle, [bolt, nut]).  
assembly(frontframe, [fork, handles]).
```

# Bike as a Tree



# Program

Write a program that, given a part, will list all the basic parts required to construct it.

Idea:

1. If the part is a basic part then nothing more is required.
2. If the part is an assembly, apply the same process (of finding subparts) to each part of it.

## Predicates: `partsof`

`partsof(X, Y)`: Succeeds if `X` is a part of bike, and `Y` is the list of basic parts required to construct `X`.

- ▶ **Boundary condition. Basic part:**

```
partsof(X, [X]) :- basicpart(X) .
```

- ▶ **Assembly:**

```
partsof(X, P) :-  
    assembly(X, Subparts) ,  
    partsoflist(Subparts, P) .
```

- ▶ **Need to define `partsoflist`.**



## Predicates: `partsoflist`

- ▶ Boundary condition. List of parts for the empty list is empty:

```
partsoflist([], []).
```

- ▶ Recursive case. For a nonempty list, first find `partsof` of the head, then recursively call `partsoflist` on the tail of the list, and glue the obtained lists together:

```
partsoflist([P|Tail], Total) :-  
    partsof(P, Headparts),  
    partsoflist(Tail, Tailparts),  
    append(Headparts, Tailparts, Total).
```

▶ The same example using accumulators

## Finding Parts

```
?- partsof(bike,Parts).
```

```
Parts=[spoke,rim,gears,bolt,nut,spoke,rim,  
       gears,bolt,nut,rearframe,fork,handles] ;
```

No

```
?- partsof(wheel,Parts).
```

```
Parts=[spoke, rim, gears, bolt, nut] ;
```

No

# Using Intermediate Results

Frequent situation:

- ▶ Traverse a PROLOG structure.
- ▶ Calculate the result which depends on what was found in the structure.
- ▶ At intermediate stages of the traversal there is an intermediate value for the result.

Common technique:

- ▶ Use an argument of the predicate to represent the "answer so far".
- ▶ This argument is called an accumulator.

# Length of a List without Accumulators

## Example

`listlen(L,N)` succeeds if the length of list `L` is `N`.

- ▶ **Boundary condition.** The empty list has length 0:

```
listlen([],0).
```

- ▶ **Recursive case.** The length of a nonempty list is obtained by adding one to the length of the tail of the list.

```
listlen([H|T],N):-  
    listlen(T,N1),  
    N is N1 + 1.
```

# Length of a List with an Accumulator

## Example

`listlenacc(L, A, N)` succeeds if the length of list `L`, when added the number `A`, is `N`.

- ▶ **Boundary condition.** For the empty list, the length is whatever has been accumulated so far, i.e. `A`:

```
lenacc([], A, A).
```

- ▶ **Recursive case.** For a nonempty list, add 1 to the accumulated amount given by `A`, and recur to the tail of the list with a new accumulator value `A1`:

```
lenacc([H|T], A, N) :-  
    A1 is A + 1,  
    lenacc(T, A1, N).
```

# Length of a List with an Accumulator, Cont.

## Example

Complete program:

```
listlen(L,N):-lenacc(L,0,N).
```

```
lenacc([],A,A).
```

```
lenacc([H|T],A,N):-
```

```
    A1 is A + 1,
```

```
    lenacc(T,A1,N).
```

# Computing List Length

Example (Version without Accumulator)

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```
listlen([a,b,c],N).
```



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listlen([a,b,c],N).
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```
listlen([b,c],N1), N is N1 + 1.
```

# Computing List Length

## Example (Version without Accumulator)

```
listlen([a,b,c],N) .
```

```
listlen([b,c],N1), N is N1 + 1.
```

```
listlen([c],N2), N1 is N2 + 1, N is N1 + 1.
```

# Computing List Length

## Example (Version without Accumulator)

```
listlen([a,b,c],N).
```

```
listlen([b,c],N1), N is N1 + 1.
```

```
listlen([c],N2), N1 is N2 + 1, N is N1 + 1.
```

```
listlen([],N3), N2 is N3 + 1, N1 is N2 + 1, N  
is N1 + 1.
```

# Computing List Length

## Example (Version without Accumulator)

`listlen([a,b,c],N)` .

`listlen([b,c],N1)`, N is  $N1 + 1$ .

`listlen([c],N2)`, N1 is  $N2 + 1$ , N is  $N1 + 1$ .

`listlen([],N3)`, N2 is  $N3 + 1$ , N1 is  $N2 + 1$ , N  
is  $N1 + 1$ .

N2 is  $0 + 1$ , N1 is  $N2 + 1$ , N is  $N1 + 1$ .

# Computing List Length

## Example (Version without Accumulator)

`listlen([a,b,c],N)` .

`listlen([b,c],N1)`, N is  $N1 + 1$ .

`listlen([c],N2)`, N1 is  $N2 + 1$ , N is  $N1 + 1$ .

`listlen([],N3)`, N2 is  $N3 + 1$ , N1 is  $N2 + 1$ , N  
is  $N1 + 1$ .

N2 is  $0 + 1$ , N1 is  $N2 + 1$ , N is  $N1 + 1$ .

N1 is  $1 + 1$ , N is  $N1 + 1$ .

# Computing List Length

## Example (Version without Accumulator)

`listlen([a,b,c],N)` .

`listlen([b,c],N1)`, N is  $N1 + 1$ .

`listlen([c],N2)`, N1 is  $N2 + 1$ , N is  $N1 + 1$ .

`listlen([],N3)`, N2 is  $N3 + 1$ , N1 is  $N2 + 1$ , N  
is  $N1 + 1$ .

N2 is  $0 + 1$ , N1 is  $N2 + 1$ , N is  $N1 + 1$ .

N1 is  $1 + 1$ , N is  $N1 + 1$ .

N is  $2 + 1$ .

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## Example (Version without Accumulator)

`listlen([a,b,c],N)` .

`listlen([b,c],N1)`, N is  $N1 + 1$ .

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`listlen([],N3)`, N2 is  $N3 + 1$ , N1 is  $N2 + 1$ , N  
is  $N1 + 1$ .

N2 is  $0 + 1$ , N1 is  $N2 + 1$ , N is  $N1 + 1$ .

N1 is  $1 + 1$ , N is  $N1 + 1$ .

N is  $2 + 1$ .

N = 3

# Computing List Length

## Example (Version with an Accumulator)

```
listlen([a,b,c], 0, N) .
```



# Computing List Length

## Example (Version with an Accumulator)

```
listlen([a,b,c], 0, N) .
```

```
listlen([b,c], 1, N) .
```

# Computing List Length

## Example (Version with an Accumulator)

```
listlen([a,b,c], 0, N) .
```

```
listlen([b,c], 1, N) .
```

```
listlen([c], 2, N) .
```

# Computing List Length

## Example (Version with an Accumulator)

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listlen([a,b,c], 0, N) .
```

```
listlen([b,c], 1, N) .
```

```
listlen([c], 2, N) .
```

```
listlen([], 3, N) .
```

# Computing List Length

## Example (Version with an Accumulator)

```
listlen([a,b,c], 0, N) .
```

```
listlen([b,c], 1, N) .
```

```
listlen([c], 2, N) .
```

```
listlen([], 3, N) .
```

$N = 3$

# List as an Accumulator

- ▶ Accumulators need not be integers.
- ▶ If a list is to be produced as a result, an accumulator will hold a list produced so far.
- ▶ Wasteful joining of structures avoided.

## Example (Reversing Lists)

```
reverse(List, Rev) :- rev_acc(List, [], Rev).
```

```
rev_acc([], Acc, Acc).
```

```
rev_acc([X|T], Acc, Rev) :-  
    rev_acc(T, [X|Acc], Rev).
```

# Bicycle Factory

Recall how parts of bike were found. [▶ Inventory example](#)

`partsof` list has to find the parts coming from the list

`[wheel, wheel, frame]`:

- ▶ **Find** parts of `frame`.
- ▶ **Append** them to `[]` to find parts of `[frame]`.
- ▶ **Find** parts of `wheel`.
- ▶ **Append** them to the parts of `[frame]` to find parts of `[wheel, frame]`.
- ▶ **Find** parts of `wheel`.
- ▶ **Append** them to the parts of `[wheel, frame]` to find parts of `[wheel, wheel, frame]`.

Wasteful!

# Bicycle Factory

Improvement idea: Get rid of append.

Use accumulators.

```
partsof(X,P):-partsacc(X,[],P).
partsacc(X,A,[X|A]):-basicpart(X).
partsacc(X,A,P):-
    assembly(X,Subparts),
    partsacclist(Subparts,A,P).
partsacclist([],A,A).
partsacclist([P|Tail],A,Total):-
    partsacc(P,A,Headparts),
    partsacclist(Tail,Headparts,Total).

partsacc(X,A,P): parts of X, when added to A, give P.
```

# Difference Structures

Compute parts of wheel without and with accumulator:

## Example (Without Accumulator)

```
?- partsof(wheel,P).
```

```
X = [spoke, rim, gears, bolt, nut] ;
```

```
No
```

## Example (With Accumulator)

```
?- partsof(wheel,P).
```

```
X = [nut, bolt, gears, rim, spoke] ;
```

```
No
```

Reversed order.



# Difference Structures

How to avoid wasteful work and retain the original order at the same time?

Difference structures.

# Open Lists and Difference Lists

- ▶ Consider the list  $[a, b, c | X]$ .
- ▶ The structure of the list is known up to a point.
- ▶ If, at some point,  $X$  is unbound then we have an **open list**.
- ▶ Informally,  $X$  is called a “hole”.

# Open Lists and Difference Lists

- ▶ Unify  $X$  with  $[d, e]$ :

`?- List=[a,b,c|X], X=[d,e].`

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```
List=[a,b,c,d,e]
```

# Open Lists and Difference Lists

- ▶ Unify  $X$  with  $[d, e]$ :

```
?- List=[a,b,c|X], X=[d,e].
```

```
List=[a,b,c,d,e]
```

- ▶ We started with an open list and “filled” in the hole with the structure.

# Open Lists and Difference Lists

- ▶ The result of filling in the hole in an open list with a “proper” list is a “proper” list.
- ▶ What happens if we instantiate the hole with an open list?

# Open Lists and Difference Lists

- ▶ The result of filling in the hole in an open list with a “proper” list is a “proper” list.
- ▶ What happens if we instantiate the hole with an open list?
- ▶ The result will be an open list again:

?- List=[a,b,c|X], X=[d,e|Y].

?- List=[a,b,c,d,e|Y].

# Open Lists and Difference Lists

- ▶ Filling in the hole with a proper list, again:
- ▶ ?- List=[a,b,c|X], X=[d,e].
- ▶ ?- List=[a,b,c,d,e].
- ▶ Is not it the same as `append([a,b,c],[d,e],List)`?



## open\_append

- ▶ We can define `append` in terms of “hole filling”.
- ▶ Assume the first list is given as an open list.
- ▶ Define a predicate that fills in the hole with the second list.
- ▶ A naive and limited way of doing this:

```
open_append([H1, H2, H3 | Hole], L2) :- Hole=L2.
```

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```

```
?- X=[a, b, c | Ho], open_append(X, [d, e]).
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```

```
    X=[a, b, c, d, e]
```

```
    Ho=[d, e]
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```

```
    X=[a, b, c, d, e]
```

```
    Ho=[d, e]
```

- ▶ Improvement is needed: This version assumes having a list with three elements and the hole.

# Improvement Idea

- ▶ One often wants to say about open lists something like  
*“take the open list and fill in the hole with ...”*
- ▶ Hence, one should know **both** an open list and a hole.
- ▶ Idea for list representation: Represent a list as an open list **together** with the hole.
- ▶ Such a representation is called a **difference list**.
- ▶ Example: The difference list representation of the list  $[a, b, c]$  is the pair of terms  $[a, b, c | X]$  and  $X$ .

## diff\_append

- ▶ Difference append:

```
diff_append(OpenList, Hole, L2) :-Hole=L2.
```

## diff\_append

► Difference append:

```
diff_append(OpenList, Hole, L2) :- Hole = L2.
```

```
?- X = [a, b, c | Ho], diff_append(X, Ho, [d, e]).
```

## diff\_append

► Difference append:

```
diff_append(OpenList, Hole, L2) :- Hole=L2.
```

```
?- X=[a,b,c|Ho], diff_append(X,Ho,[d,e]).
```

```
    X=[a,b,c,d,e]
```

```
    Ho=[d,e]
```



## diff\_append

- ▶ **Difference append:**

```
diff_append(OpenList, Hole, L2) :- Hole=L2.
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?- X=[a,b,c|Ho], diff_append(X, Ho, [d,e]).
```

```
    X=[a,b,c,d,e]
```

```
    Ho=[d,e]
```

- ▶ **Compare to the open\_append:**

```
open_append([H1,H2,H3|Hole], L2) :- Hole=L2.
```

## diff\_append

- ▶ **Difference append:**

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diff_append(OpenList, Hole, L2) :- Hole=L2.
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?- X=[a,b,c|Ho], diff_append(X, Ho, [d,e]).
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    X=[a,b,c,d,e]
```

```
    Ho=[d,e]
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- ▶ **Compare to the open\_append:**

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open_append([H1,H2,H3|Hole], L2) :- Hole=L2.
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?- X=[a,b,c|Ho], open_append(X, [d,e]).
```

```
    X=[a,b,c,d,e]
```

```
    Ho=[d,e]
```

# Difference Lists

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# Difference Lists

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- ▶ From the pair  $[a, b, c | H_0]$  and  $H_0$  we are interested in  $[a, b, c]$ .
- ▶ “Subtracting” the hole  $H_0$  from the open list  $[a, b, c | H_0]$ .

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- ▶  $[a, b, c | H_0] - H_0$ .

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- ▶ From the pair  $[a, b, c | H_0]$  and  $H_0$  we are interested in  $[a, b, c]$ .
- ▶ “Subtracting” the hole  $H_0$  from the open list  $[a, b, c | H_0]$ .
- ▶  $[a, b, c | H_0] - H_0$ .
- ▶ The  $-$  has no interpreted meaning. Instead one could define any operator to use there.

## diff\_append. Version 2

- ▶ `diff_append(OpenList-Hole, L2) :-Hole=L2.`



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`Ho=[d,e]`

## diff\_append. Version 2

- ▶ `diff_append(OpenList-Hole, L2) :- Hole=L2.`  
`?- X=[a,b,c|Ho]-Ho, diff_append(X, [d,e]).`  
`X=[a,b,c,d,e]-[d,e]`  
`Ho=[d,e]`
- ▶ Has to be improved again: We are not interested in the “filled hole” in the instantiation of `X` hanging around.

## diff\_append. Version 3

- ▶ Let `diff_append` return the open list part of the first argument:

```
diff_append(OpenList-Hole, L2, OpenList) :-Hole=L2.
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```
diff_append(OpenList-Hole, L2, OpenList) :- Hole=L2.  
?- X=[a,b,c|Ho]-Ho, diff_append(X, [d,e], Ans).
```

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```
diff_append(OpenList-Hole, L2, OpenList) :-Hole=L2.
```

```
?- X=[a,b,c|Ho]-Ho, diff_append(X, [d,e], Ans).
```

```
  X=[a,b,c,d,e]-[d,e]
```

```
  Ho=[d,e]
```

```
  Ans=[a,b,c]
```

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```
diff_append(OpenList-Hole, L2, OpenList) :- Hole=L2.  
?- X=[a,b,c|Ho]-Ho, diff_append(X, [d,e], Ans).  
   X=[a,b,c,d,e]-[d,e]  
   Ho=[d,e]  
   Ans=[a,b,c]
```

- ▶ It is better now. `Ans` looks as we would like to.

## diff\_append. Version 3

- ▶ Let `diff_append` return the open list part of the first argument:

```
diff_append(OpenList-Hole, L2, OpenList) :-Hole=L2.  
?- X=[a,b,c|Ho]-Ho, diff_append(X, [d,e], Ans).  
   X=[a,b,c,d,e]-[d,e]  
   Ho=[d,e]  
   Ans=[a,b,c]
```

- ▶ It is better now. `Ans` looks as we would like to.
- ▶ Still, there is a room for improvement: The `diff_append`
  - ▶ takes a difference list as its first argument,
  - ▶ a proper list as its second argument, and
  - ▶ returns a proper list.
- ▶ Let's make it more uniform.



## diff\_append. Version 3

- ▶ **Better, but not the final approximation:** `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

## diff\_append. Version 3

- ▶ **Better, but not the final approximation:** `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
    diff_append(X, [d,e|Ho1]-Ho1,Ans) .
```

## diff\_append. Version 3

- ▶ Better, but not the final approximation: `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1,Ans) .  
X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
Ho=[d,e|Ho1]  
Ans=[a,b,c|Ho1]
```

## diff\_append. Version 3

- ▶ Better, but not the final approximation: `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1,Ans) .  
X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
Ho=[d,e|Ho1]  
Ans=[a,b,c|Ho1]
```

- ▶ We have returned an open list but we want a difference list.

## diff\_append. Version 3

- ▶ Better, but not the final approximation: `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1,Ans) .  
X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
Ho=[d,e|Ho1]  
Ans=[a,b,c|Ho1]
```

- ▶ We have returned an open list but we want a difference list.
- ▶ The first list has gained the hole of the second list.

## diff\_append. Version 3

- ▶ Better, but not the final approximation: `diff_append` takes two difference lists and returns an open list:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X,[d,e|Ho1]-Ho1,Ans).  
X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
Ho=[d,e|Ho1]  
Ans=[a,b,c|Ho1]
```

- ▶ We have returned an open list but we want a difference list.
- ▶ The first list has gained the hole of the second list.
- ▶ All we need to ensure is that we return the hole of the second list.

## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2) :-Hole1=OpenList2.
```

## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2):-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X,[d,e|Ho1]-Ho1,Ans).
```



## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2) :-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1,Ans) .  
   X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
   Ho=[d,e|Ho1]  
   Ans=[a,b,c|Ho1]-Ho1
```

## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2) :- Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1, Ans) .  
   X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
   Ho=[d,e|Ho1]  
   Ans=[a,b,c|Ho1]-Ho1
```

- ▶ We have returned an difference list.

## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2) :- Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
   diff_append(X, [d,e|Ho1]-Ho1, Ans) .  
   X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
   Ho=[d,e|Ho1]  
   Ans=[a,b,c|Ho1]-Ho1
```

- ▶ We have returned an difference list.
- ▶ Now we can recover the proper list we want this way:

## diff\_append. Version 3

- ▶ Return the hole of the second list as well:

```
diff_append(OpenList1-Hole1,  
            OpenList2-Hole2,  
            OpenList1-Hole2):-Hole1=OpenList2.
```

```
?- X=[a,b,c|Ho]-Ho,  
    diff_append(X,[d,e|Ho1]-Ho1,Ans).  
    X=[a,b,c,d,e|Ho1]-[d,e|Ho1]  
    Ho=[d,e|Ho1]  
    Ans=[a,b,c|Ho1]-Ho1
```

- ▶ We have returned an difference list.
- ▶ Now we can recover the proper list we want this way:

```
?- X=[a,b,c|Ho]-Ho,  
    diff_append(X,[d,e|Ho1]-Ho1,Ans-[]).  
    Ans=[a,b,c]
```

## diff\_append. Version 4

- ▶ `diff_append` **can be made more compact:**

```
diff_append(OpenList1-Hole1,  
           Hole1-Hole2,  
           OpenList1-Hole2).
```

## diff\_append. Usage

- ▶ Add an element at the end of a list:

```
add_to_back(L-H, El, Ans) :-  
    diff_append(L-H, [El | H1]-H1, Ans-[]).
```

## diff\_append. Usage

- ▶ Add an element at the end of a list:

```
add_to_back(L-H, El, Ans) :-  
    diff_append(L-H, [El|H1]-H1, Ans-[]).  
?- add_to_back([a,b,c|H]-H,e,Ans).
```

## diff\_append. Usage

- ▶ Add an element at the end of a list:

```
add_to_back(L-H, El, Ans) :-  
    diff_append(L-H, [El|H1]-H1, Ans-[]).  
?- add_to_back([a,b,c|H]-H,e,Ans).  
   H = [e]  
   Ans = [a,b,c,e]
```



# Difference Structures

Both accumulators and difference structures use two arguments to build the output structure.

**Assumulators:** the “result so far” and the “final result”.

**Difference structures:** the (current approximation of the) “final result” and the “hole in there where the further information can be put”.

# Bicycle Factory

Use holes.

```
partsof(X,P):-partshole(X,P-Hole),Hole=[].
partshole(X,[X|Hole]-Hole):-basicpart(X).
partshole(X,P-Hole):-
    assembly(X,Subparts),
    partsholelist(Subparts,P-Hole).
partsholelist([],Hole-Hole).
partsholelist([P|Tail],Total-Hole):-
    partshole(P,Total-Hole1),
    partsholelist(Tail,Hole1-Hole).
```

## Bicycle Factory. Detailed View

```
partsof(X,P):-partshole(X,P-Hole),Hole=[].
```

- ▶ `partshole(X,P-Hole)` builds the result in the second argument `P` and returns in `Hole` a variable.
- ▶ Since `partsof` calls `partshole` only once, it is necessary to terminate the difference list by instantiating `Hole` with `[]`. (Filling the hole.)
- ▶ Alternative definition of `partsof`:  

```
partsof(X,P):-partshole(X,P-[]).
```

It ensures that the very last hole is filled with `[]` even before the list is constructed.

## Bicycle Factory. Detailed View

```
partshole(X, [X|Hole]-Hole):-basicpart(X).
```

- ▶ It returns a difference list containing the object (basic part) in the first argument.
- ▶ The hole remains open for further instantiations.

## Bicycle Factory. Detailed View

```
partshole(X,P-Hole):-  
    assembly(X,Subparts),  
    partsholelist(Subparts,P-Hole).
```

- ▶ Finds the list of subparts.
- ▶ Delegates the traversal of the list to `partsholelist`.
- ▶ The difference list `P-Hole` is passed to `partsholelist`.

## Bicycle Factory. Detailed View

```
partsholelist([P|Tail],Total-Hole):-  
    partshole(P,Total-Hole1),  
    partsholelist(Tail,Hole1-Hole).
```

- ▶ `partshole` starts building the `Total` list, partially filling it with the parts of `P`, and leaving a hole `Hole1` in it.
- ▶ `partsholelist` is called recursively on the `Tail`. It constructs the list `Hole1` partially, leaving a hole `Hole` in it.
- ▶ Since `Hole1` is shared between `partshole` and `partsholelist`, after getting instantiated in `partsholelist` it gets also instantiated in `partshole`.
- ▶ Therefore, at the end `Total` consists of the portion that `partshole` constructed, the portion of `Hole1` `partsholelist` constructed, and the hole `Hole`.