Computer Algebra Systems

(Topics)

1. **GAP (Groups, Algorithms and Programming)** is a system for computational discrete algebra, with particular emphasis on computational group theory. It is used in research and teaching for studying groups and their representations, rings, vector spaces, algebras, combinatorial structures, and more (http://www.gap-system.org/).

2. **Magma** is a computer algebra system designed to solve problems in group theory, number theory, linear algebra, geometry, representation theory, and combinatorics. It provides a mathematically rigorous environment for computing with algebraic, number-theoretic, combinatorial, and geometric objects (not free, http://magma.maths.usyd.edu.au/magma/).

3. **PARI/GP** is a widely used computer algebra system designed for fast computations in number theory (factorizations, algebraic number theory, elliptic curves), but also contains a large number of other useful functions to compute with mathematical entities such as matrices, polynomials, power series, and algebraic numbers (http://pari.math.u-bordeaux.fr/).

4. **Sage** is a free open-source mathematics software system licensed under the GPL. It combines the power of many existing open-source packages into a common Python-based interface (http://www.sagemath.org/).

5. **Singular (and a Short Introduction to Gröbner Bases).** How can we find the solutions of a system of polynomial equations? The answer can be given by means of Gröbner bases, and one of the fastest systems to compute those is Singular. Singular is a computer algebra system for polynomial computations, with special emphasis on commutative and non-commutative algebra, algebraic geometry, and singularity theory (http://www.singular.uni-kl.de/).

6. **Axiom** is a free general purpose computer algebra system. It defines a strongly typed, mathematically correct type hierarchy (http://www.axiom-developer.org/).

7. **Differential Equations with Computer Algebra.** Several algorithms and methods of how to treat differential equations in a symbolic way are discussed, and several computer algebra systems are compared with respect to their abilities in this field.

8. **Algebraic Numbers.** It is well known that in general the solutions of a polynomial equation of degree 5 or higher cannot be expressed with radicals. So how can we deal with such quantities *symbolically*? Which functionality do computer algebra systems provide to deal with such numbers?

9. **Quantifier Elimination with CAD.** Cylindrical Algebraic Decomposition (CAD) is an algorithm for solving problems that are given in terms of polynomial inequalities. Some computer algebra systems (e.g., Mathematica) use it to perform quantifier elimination.

10. **Decidability Questions.** When doing symbolic computations, we often encounter undecidable problems. For example, equality test and zero test in rather simple domains are undecidable.