

■ GCD

```
G[a_, 0] := a;  
G[a_, b_] := G[b, Mod[a, b]];
```

■ GCD with Trace

```
G[a_, 0] := (Print["<", a, ", ", 0, ""]; a);  
G[a_, b_] := (Print["<", a, ", ", b, ""]; G[b, Mod[a, b]]);
```

■ GCD test

```
G[15, 10]  
  
<15,10>  
<10,5>  
<5,0>  
  
5
```

```
G[115 878, 5 546 511]  
  
<115 878,5 546 511>  
<5 546 511,115 878>  
<115 878,100 245>  
<100 245,15 633>  
<15 633,6447>  
<6447,2739>  
<2739,969>  
<969,801>  
<801,168>  
<168,129>  
<129,39>  
<39,12>  
<12,3>  
<3,0>  
  
3
```

■ Simple Sorting

```
In[1]:= Clear[MySort]  
In[2]:= MySort[{L1___, a_, b_, L2___}] /; (a > b) := MySort[{L1, b, a, L2}]  
In[3]:= MySort[L_] := L
```

Test Simple SortingIn[4]:= **MySort**[{}]

Out[4]= {}

In[5]:= **MySort**[{1, 2, 3}]

Out[5]= {1, 2, 3}

In[6]:= **MySort**[{4, 5, 1, 2, 3}]

Out[6]= {1, 2, 3, 4, 5}

In[7]:= **MySort**[{4, 5, 3, 1, 2, 5, 3}]

Out[7]= {1, 2, 3, 3, 4, 5, 5}

■ Simple Sorting with TraceIn[17]:= **Clear**[MySort]In[18]:= **MySort**[{L1____, a_, b_, L2____}] /; (a > b) := (Print[{L1, b, a, L2}]; **MySort**[{L1, b, a, L2}])In[19]:= **MySort**[L_] := L**■ Test Simple Sorting with Trace**In[20]:= **MySort**[{}]

Out[20]= {}

In[21]:= **MySort**[{1, 2, 3}]

Out[21]= {1, 2, 3}

In[22]:= **MySort**[{4, 5, 1, 2, 3}]

{4, 1, 5, 2, 3}

{1, 4, 5, 2, 3}

{1, 4, 2, 5, 3}

{1, 2, 4, 5, 3}

{1, 2, 4, 3, 5}

{1, 2, 3, 4, 5}

Out[22]= {1, 2, 3, 4, 5}

In[23]:= **MySort**[{4, 5, 3, 1, 2, 5, 3}]

```

{4, 3, 5, 1, 2, 5, 3}
{3, 4, 5, 1, 2, 5, 3}
{3, 4, 1, 5, 2, 5, 3}
{3, 1, 4, 5, 2, 5, 3}
{1, 3, 4, 5, 2, 5, 3}
{1, 3, 4, 2, 5, 5, 3}
{1, 3, 2, 4, 5, 5, 3}
{1, 2, 3, 4, 5, 5, 3}
{1, 2, 3, 4, 5, 3, 5}
{1, 2, 3, 4, 3, 5, 5}
{1, 2, 3, 3, 4, 5, 5}
Out[23]= {1, 2, 3, 3, 4, 5, 5}

```

■ SumFirst

```

In[8]:= Clear[SumFirst];
In[9]:= SumFirst[0] := 0;
In[10]:= SumFirst[n_] := n + SumFirst[n - 1]

```

■ Test SumFirst

```

In[14]:= SumFirst[3]
Out[14]= 6

```

■ SumFirst Tail Recursive

```

In[11]:= Clear[SumFirstT];
In[12]:= SumFirstT[m_, 0] := m;
In[13]:= SumFirstT[m_, n_] := SumFirstT[m + n, n - 1]

```

■ Test SumFirst Tail Recursive

```

In[15]:= SumFirstT[0, 3]
Out[15]= 6

```

■ SumFirst with Trace

```

In[24]:= Clear[SumFirst];
In[25]:= SumFirst[0] := 0;
In[26]:= SumFirst[n_] := (Print[n]; n + SumFirst[n - 1])

```

Test SumFirst with Trace

```
In[27]:= SumFirst[3]
```

```
3
```

```
2
```

```
1
```

```
Out[27]= 6
```

■ SumFirst Tail Recursive with Trace

```
In[32]:= Clear[SumFirstT];
```

```
In[33]:= SumFirstT[m_, 0] := m;
```

```
In[34]:= SumFirstT[m_, n_] := (Print[m, ", ", n]; SumFirstT[m + n, n - 1])
```

■ Test SumFirst Tail Recursive with Trace

```
In[35]:= SumFirstT[0, 3]
```

```
0, 3
```

```
3, 2
```

```
5, 1
```

```
Out[35]= 6
```