Logic 1, WS 2012. Homework 1, given Oct 17, due Oct 24

1. Define the meta-function $L[\varphi]$ which gives the lenght of a propositional formula.
Hint: use the induction principle suggested by the definition of propositional logic formulae.

2. Using the definition above and the definition of the function $D[\varphi]$ (depth of a propositional formula) given in the lecture, prove that $D[\varphi] \leq L[\varphi]$ for any propositional formula φ.
Hint: use the induction principle suggested by the definition of propositional logic formulae.

3. Prove that for any propositional formulae φ, ψ, if $\varphi \models \psi$ and $\psi \models \varphi$, then $\varphi \equiv \psi$.
(See the style used in the lecture for proving the opposite implication.)

4. Prove that for any propositional formulae $\varphi, \psi : \varphi \models \psi$ iff $\varphi \Leftrightarrow \psi$ is valid.
(The proof may be informal.)

5. Prove that for any propositional formulae $\varphi_1, \varphi_2, \ldots, \varphi_n, \psi$, if $\varphi_1, \varphi_2, \ldots, \varphi_n \models \psi$, then $(\varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_n) \Rightarrow \psi$ is valid.
(See the style used in the lecture for proving the opposite implication.)