to be prepared for 15.10.2013

Exercise 1. The idea of Karatsuba can also be applied to the multiplication of polynomials. Determine the number of multiplications necessary for multiplying the polynomials

$$a_0x^3 + a_1x^2 + a_2x + a_3 \text{ und } b_1x^2 + b_2x + b_3$$

both with the classical and with the Karatsuba method.

Exercise 2. Consider the integers $a = 215712,\ b = 739914$. Determine the gcd of a and b and integers s, t such that $\gcd(a, b) = sa + tb$.

Exercise 3. Use the Euclidean algorithm for finding the positive integer solutions of the equation

$$34x + 38y = 1000.$$

Exercise 4. Consider the polynomials

$$\begin{aligned} f &= x^6 - 2x^5 - x^4 - 4x^3 - 5x^2 - 2x - 3 \\ g &= 3x^6 - x^5 + 4x^4 - 2x^3 + 2x^2 - x + 1. \end{aligned}$$

f and g are elements of the ring $\mathbb{Z}[x]$. Apply the extended Euclidean algorithm in $\mathbb{Q}[x]$ to compute $\gcd(f, g)$ as a linear combination of f and g. Use a computeralgebra system of your choice.

Exercise 5. Consider the polynomials $U(x), V(x)$ computed by the extended Euclidean algorithm for the input polynomials $u(x), v(x)$ over $\mathbb{Q}$, i.e., $\gcd(u, v) = Uu+Vv$. Prove that, if $u = x^m - 1$ and $v = x^n - 1$, then the extended Euclidean algorithm provides polynomials U, V with integer coefficients. Find U and V when $u = x^{23} - 1$ and $v = x^{18} - 1$.