to be prepared for 29.10.2013

Exercise 14. Prove that every Euclidean domain is a principal ideal domain, and that every principal ideal domain is a unique factorization domain.

Exercise 15. Consider the two polynomials over $\mathbb{Z}$

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2 \ . \end{aligned}$$

Compute $\gcd(f(x), g(x))$

1. by passing to the quotient field;
2. by a polynomial remainder sequence.

Exercise 16. Consider the two polynomials over $\mathbb{Z}$

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2 \ . \end{aligned}$$

Compute $\gcd(f(x), g(x))$ by the modular algorithm.

Exercise 17. Compute the gcd of the bivariate integer polynomials

$$\begin{aligned} f(x,y) &= y^6 + xy^5 + x^3y - xy + x^4 - x^2, \\ g(x,y) &= xy^5 - 2y^5 + x^2y^4 - 2xy^4 + xy^2 + x^2y \end{aligned}$$

by the subresultant algorithm.

Exercise 18. Consider the bivariate polynomials

$$\begin{aligned} f(x,y) &= x^2y^3 - 5xy^3 + 6y^3 - 6xy^2 + 18y^2 + 2xy - 4y - 12, \\ g(x,y) &= x^2y^3 + 3xy^3 - 10y^3 - 6xy^2 - 30y^2 - 4xy + 8y + 24. \end{aligned}$$

Compute the gcd of f and g by the modular algorithm. Take care of leading coefficients.