to be prepared for 05.11.2013

Exercise 19. Let $\mathbb{Z}_5(\alpha)$ be the algebraic extension of $\mathbb{Z}_5$ by a root α of the irreducible polynomial x^5+4x+1. Compute the normal representation of $(ab)/c$, where $a = \alpha^3 + \alpha + 2$, $b = 3\alpha^4 + 2\alpha^2 + 4\alpha$, $c = 2\alpha^4 + \alpha^3 + 2\alpha + 1$.

Exercise 20. Consider the algebraic extension $\mathbb{Q}(\alpha)$ of $\mathbb{Q}$ with $\alpha^4+2\alpha^3+\alpha+1 = 0$. Let $u = \alpha^3 + \alpha^2 + \alpha + 1$, $v = \alpha^3 + 2\alpha^2 + \alpha - 1$. Compute the normal representation of $(uv)^{-1}$.

Exercise 21. Choose two different irreducible polynomials for constructing the Galois field $GF(2^3)$ in two essentially different ways. Demonstrate that your resulting fields are actually isomorphic.

Exercise 22. Prove the following statement. *If k is a finite field then k is a simple extension of some $\mathbb{Z}_p$.*

Exercise 23. How many factors does $u(x) = x^4 + 1$ have in $\mathbb{Z}_p[x]$, p a prime? (Hint: Consider the cases $p = 2,\ 8k + 1,\ 8k + 3,\ 8k + 5,\ 8k + 7$ separately).

Exercise 24. Give an example of a polynomial $f \in k[x]$ that is irreducible with multiple roots.