to be prepared for 14.10.2014

Exercise 1. Consider the polynom

$$f = x^5 - x^4 + x^3 - x^2 + x - 2.$$

Use a computer algebra system to perform the following tasks.

1. Compute the zeros of f numerically. You have influence on floating point precision, if you want to.
2. Generate a picture of the graph of the polynomial function $x \mapsto f(x)$ on an interval $[a, b]$. Choose a and b in such a way that you can 'see' the real zeros of f.
3. Compute the zeros of f symbolically. Which output comes from your computer algebra system?
4. Compute the zeros of the polynomial

$$f = 2x^2 + 2x^3 + 2x^4 + x^5 - x^6 + 3x + 1.$$

Exercise 2. Given the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 & 4 \end{pmatrix}.$$

Compute all solutions of the linear system $A(x_1, x_2, x_3, x_4, x_5)^T = (1, 2, 3, 4, 5)^T$. Do it with the aid of a computer algebra system of your choice.

Exercise 3. In your favorite computer algebra system find out about possibilities for solving systems of polynomial equations.

1. Consider the system of equations

$$\begin{aligned} 2x^4 - 3x^2y + y^4 - 2y^3 + y^2 &= 0 \\ 4x^3 - 3xy &= 0 \\ 4y^3 - 3x^2 - 6y^2 + 2 &= 0. \end{aligned}$$

   Compute all solutions.

2. The same for

$$\begin{aligned} 1 + 8xy + 2y^2 + 8xy^3 + y^4 - 16x^2 &= 0 \\ 8x + 4y + 24xy^2 + 4y^3 &= 0 \\ 8y + 8y^3 - 32x &= 0. \end{aligned}$$

Exercise 4. An integral domain is a commutative ring $D \neq \{0\}$ without zero divisors, that means, $rs = 0 \Rightarrow r = 0 \vee s = 0$ $(\forall r, s \in D)$. Give a proof for the following statement.

1. If D is an integral domain, then also the polynomial ring $D[x]$.
2. Derive from this that - for arbitrary fields k - the ring $k[x_1, \ldots, x_n]$ is an integral domain.
3. Give similar arguments for the ring $D[[x]]$ of formal power series.