to be prepared for 21.10.2014

Exercise 5. Consider the integers $a = 215712,\ b = 739914$. Determine the gcd of a and b and integers s, t such that $\gcd(a, b) = sa + tb$.

Exercise 6. Consider the polynomials

$$\begin{aligned} f &= x^6 - 2x^5 - x^4 - 4x^3 - 5x^2 - 2x - 3 \\ g &= 3x^6 - x^5 + 4x^4 - 2x^3 + 2x^2 - x + 1. \end{aligned}$$

f and g are elements of the ring $\mathbb{Z}[x]$. Apply the extended Euclidean algorithm in $\mathbb{Q}[x]$ to compute $\gcd(f, g)$ as a linear combination of f and g. Use a computeralgebra system of your choice.

Exercise 7. Use Gröbner bases for solving over $\mathbb{C}$:

$$\begin{aligned} f_1(x, y, z) &= xz - xy^2 - 4x^2 - \frac{1}{4} = 0, \\ f_2(x, y, z) &= y^2 z + 2x + \frac{1}{2} = 0, \\ f_3(x, y, z) &= x^2 z + y^2 + \frac{1}{2}x = 0. \end{aligned}$$

Exercise 8.

1. Explain why the Euclidean algorithm can be considered as a special case of Gröbner base algorithm.
2. Considering a system of linear equations in indeterminates $x_1, \ldots, x_n$, discuss the relation between Gauss elimination and Gröbner bases.

Exercise 9. Consider the polynomials

$$\begin{aligned} f_1(x, y) &= x^2 y + xy + 1, \\ f_2(x, y) &= y^2 + x + y \end{aligned}$$

in $\mathbb{Z}_3[x, y]$. Compute a Gröbner basis for the ideal $\langle f_1, f_2 \rangle$ w.r.t. the graduated lexicographical ordering with $x < y$. Show intermediate results.