to be prepared for 04.11.2014

Exercise 15. Let $\longrightarrow$ be a Noetherian reduction relation. Prove that $\longrightarrow$ is confluent if and only if it is locally confluent. (Consider Definition 2.3.12.)

Exercise 16. Give an example of a locally confluent reduction relation which is not confluent.

Exercise 17. Let K be an algebraically closed field, n a positive integer, H a hypersurface in $\mathbb{A}^n(K)$, the affine space of dimension n over K. Let $f \in K[x_1, \ldots, x_n] \setminus K$ be a defining polynomial of H, i.e.,

$$H = \{(a_1, \ldots, a_n) \mid f(a_1, \ldots, a_n) = 0\}.$$

Let $f = f_1^{m_1} \cdots f_r^{m_r}$ be the factorization of f into irreducible factors. Let I be the ideal of polynomials in $K[x_1, \ldots, x_n]$ vanishing on H. Show that $I = \langle f_1 \cdots f_r \rangle$.

Exercise 18. Try to give an answer to the following problem:
Given two ideals $I, J \subseteq k[x_1, \ldots, x_n]$ in terms of generators $I = \langle f_1, \ldots, f_r \rangle$, $J = \langle g_1, \ldots, g_s \rangle$, find generators of the intersection $I \cap J$.