to be prepared for 25.11.2014

Exercise 29. Let $I \subseteq K[x_1, \ldots, x_n]$ be an ideal and G a Gröbner basis for I. Let $g, h \in G$ with $g \neq h$. Prove the following statements.

1. If $\mathrm{lpp}(g)|\mathrm{lpp}(h)$ then $G \setminus \{h\}$ is a Gröbner basis for I.
2. If $h \longrightarrow_g h'$ then $(G \setminus \{h\}) \cup \{h'\}$ is a Gröbner basis for I.

Exercise 30. Prove that the set of polynomials $p(x, y)$ vanishing on a set $S \subseteq \mathbb{R}^2$ form an ideal in $\mathbb{R}[x, y]$. Do the polynomials NOT vanishing on $(0, 0)$ form an ideal?

Exercise 31. Consider linear polynomials in $K[x_1, \ldots, x_n]$

$$f_i = a_{i1}x_1 + \cdots + a_{in}x_n \qquad 1 \leq i \leq m$$

and let $A = (a_{ij})$ be the $m \times n$ matrix of their coefficients. Let B be the reduced row echelon matrix determined by A and let $g_1, \ldots, g_r$ be the linear polynomials coming from the nonzero rows of B. Use lex order with $x_1 > \cdots > x_n$ and show that $g_1, \ldots, g_r$ form the reduced Groebner basis of $\langle f_1, \ldots, f_m \rangle$.

Exercise 32. Use Gröbner bases to find the implicit representation of the parametrized surface

$$\begin{aligned} x &= 1 + s + t + st \\ y &= 2 + s + st + t^2 \\ z &= s + t + s^2 \end{aligned}$$