to be prepared for 09.12.2014

Exercise 33. Consider the two polynomials over $\mathbb{Z}$

$$\begin{aligned} f(x) &= 6x^5 + 2x^4 - 19x^3 - 6x^2 + 15x + 9 \\ g(x) &= 5x^4 - 4x^3 + 2x^2 - 2x - 2 \ . \end{aligned}$$

Compute $\gcd(f(x), g(x))$

1. by passing to the quotient field;
2. by a polynomial remainder sequence.

Exercise 34. Prove the following statement. *Two nonzero polynomials $f(x)$ and $g(x)$ with coefficients in a unique factorization domain are similar iff they have equal primitive part.*

Exercise 35. Compute the pseudo-quotient $q(x)$ and the pseudo-remainder $r(x)$ for the two integral polynomials

$$\begin{aligned} u(x) &= x^6 + x^5 - x^4 + 2x^3 + 3x^2 - x + 2 \quad \text{and} \\ v(x) &= 2x^3 + 2x^2 - x + 3. \end{aligned}$$

Exercise 36. Let R be an integral domain, $a, b \in R[x]$, $b \neq 0$, $m = \deg(a) \geq n = \deg(b)$, $\alpha, \beta \in R$. Prove:

$$\begin{aligned} \mathrm{pquot}(\alpha a, \beta b) &= \beta^{m-n}\alpha \cdot \mathrm{pquot}(a, b) \quad \text{and} \\ \mathrm{prem}(\alpha a, \beta b) &= \beta^{m-n+1}\alpha \cdot \mathrm{prem}(a, b). \end{aligned}$$

Exercise 37. Consider the integer polynomials

$$\begin{aligned} f_1 &= (x-5)(x-7)^3(x-10)(x^4 - 2x^3 + x^2 - 3x + 1) \\ f_2 &= (x-5)(x+7)(x-8)(x^5 + 13x^3 - 17x + 1). \end{aligned}$$

Use a primitive polynomial remainder sequence to compute the $GCD(f_1, f_2)$.

You definitely should use some computer algebra system.