Abstracts of Recent Doctoral Dissertations in Computer Algebra

Each month we are pleased to present abstracts of recent doctoral dissertations in Computer Algebra and Symbolic Computation. We encourage all recent Ph.D. graduates (and their supervisors), who have defended in the past two years, to submit their abstracts for publication in CCA.
Please send abstracts to the CCA editors `<editors_SIGSAM@acm.org>` *for consideration.*

Author: Brice Boyer
Title: Efficient matrix multiplication and design for the exact linear algebra library LinBox
Institution: Laboratoire Jean Kuntzman, Université de Grenoble.
Thesis Advisor: Jean-Guillaume Dumas
Defended: June 2012
Keywords: exact linear algebra, sparse matrix, SpMV, dense matrix, fast matrix multiplication, pebble game, schedulings, design patterns, generic mathematics library, LinBox.

Matrix multiplication is a major cornerstone in exact linear algebra: its study can concern algorithmic, complexity, design, reduction, *etc.* problems. We are interested in the few following aspects.

We first expose, in this thesis, efficient exact matrix multiplication techniques, developed for both multiplication ($A = B \times C$) and product with accumulation ($A = A + B \times C$). We set up new schedules that allow us to minimize the extra memory requirements during a Strassen–style matrix multiplication, while keeping the complexity competitive with Winograd's multiplication algorithm. In order to obtain them, we develop external tools (pebble games), tight complexity computations and new hybrid algorithms.

We then use parallel technologies (multicore CPU and GPU) in order to efficiently accelerate the sparse matrix–dense vector multiplication (SpMV) or sparse–matrix dense matrix multiplication (SpMM), crucial to *blackbox* (block) algorithms. We also set up new hybrid, environment dependant, sparse matrix formats that help yield large speed-ups. We exemplify these results by speeding up the block Winograd rank algorithm in the LinBox library.

Finally, we establish generic design methods focusing on efficiency, especially via *building block* conceptions or self-optimization. We also propose tools for improving and standardizing code quality in order to make it more sustainable and more robust. This is applied in particular to the LinBox computer algebra library.

Author: Ibrahim Adamou
Title: Curve and Surface Bisectors, and Voronoi Diagram of a family of parallel half-lines in $\mathbb{R}^3$
Institution: Universidad de Cantabria

Thesis Advisor: Laureano Gonzalez-Vega and Mario Fioravanti
Committee Members: Tomas Recio, Marie-Françoise Roy, Bert Jüttler
Defended: September 10th, 2013 *Keywords:* Bisectors, Rational Curves and Surfaces, Voronoi Diagram, Spatial Subdivision, Meshing.

This thesis has three main parts: computation of the bisectors of two curves or a point and a curve in the plane, of the bisector of two surfaces in $\mathbb{R}^3$, and of the Voronoi diagram of a finite family of parallel half lines in $\mathbb{R}^3$, with the same orientation. These subjects are closely related, and have applications in CAD/CAGD and Computational Geometry. In each of the three parts, we present algorithmic methods for computing certain representations of the geometric object of interest: the bisector curve, the bisector surface, or the Voronoi diagram.

We present a new approach to determine an algebraic parametrization (rational or non rational) of the bisector curve of two given planar rational curves. The method uses Cramer's rule and algebraic elimination steps. The method is applied, in particular, to obtain parametrizations of the bisector of two rational plane curves, when one of them is a circle or a straight line. Then, this approach is generalized to determine an algebraic parametrization of the bisector surface of two low degree rational surfaces. We show how to easily obtain parametrizations of the bisector of the following pairs of surfaces: plane-quadric, plane-torus, circular cylinder-non developable quadric, circular cylinder-torus, cylinder-cylinder, cylinder-cone and cone-cone. These parametrizations are rational in most cases. In the remaining cases, the parametrization involves one square root which is well-suited to determine a good approximation of the bisector.

In addition, we present a different approach for the bisector curve problem. This new method uses dynamic color in GeoGebra (a dynamical geometry software) for the geometric and numerical characterizations of the bisector of two curves, or a curve and a point, in the plane. Even if it does not provide an algebraic representation, the method could lead to the computation of an approximate representation of the bisector curve.

The Voronoï diagram (VD) is a fundamental data structure in computational geometry with various applications in theoretical and practical areas. We consider the VD of a set of parallel half-lines, with the same orientation, constrained to a compact domain $\mathcal{D}_0 \subset \mathbb{R}^3$, with respect to the Euclidean distance. This new kind of VD can be used to provide an efficient solution to some problems in the drilling industry. We present an efficient algorithm for computing an approximate VD, using a box subdivision process, which produces a mesh representing the topology of the VD in $\mathcal{D}_0$. The concept of minimization diagram plays an important role in the method.