Abstracts of Recent Doctoral Dissertations in Computer Algebra

Each month we are pleased to present abstracts of recent doctoral dissertations in Computer Algebra and Symbolic Computation. We encourage all recent Ph.D. graduates (and their supervisors), who have defended in the past two years, to submit their abstracts for publication in CCA.
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Title: On Isolated Singular Solutions in Polynomial System Solving
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Defended: June 2013

Nowadays, polynomial models are ubiquitous and widely applied across the engineering and sciences, such as in robotics, coding theory, optimization, mathematical biology, computer vision, game theory, statistics, machine learning, control theory, cryptography, and numerous others. A main challenge in algebra and geometry computing is to identify and tackle singular points, which naturally occur when computing the topology of implicit curves or surfaces, the intersection of parametric surfaces in geometric modeling.

A numerical approximation is usually computed to identify an isolated solution of a polynomial system. In practice, we often need to improve the quality of numerical approximations, but numerical methods such like Newton's method converge slowly at singular solutions (or not converge). On the other hand, it is well known that to certify whether a polynomial system has an isolated singular solution is an ill-posed problem, since arbitrary small perturbations of coefficients may transform the singular solution into a cluster of simple roots (or even make it disappear). Therefore, it is hardly possible to verify this problem, if not the entire computation is performed without any rounding error (exact arithmetics).

In this thesis, we first introduce the local dual space for characterizing an isolated singular solution of a polynomial system. By employing some regularization and reduction techniques, we present a novel algorithm for computing a reduced basis of such a space for the special case of breadth one. The algorithm also works for inputs only with limited accuracy, and is efficient both in time and memory use. Moreover, it leads to a parametric representation for a reduce basis (multiplicity structure) of the local dual space.

Based on such a parametric representation and presolving a regularized least squares problem, we propose a regularized Newton's method for refining an approximate singular solution of a given polynomial system. By a careful analysis, we prove the quadratic convergence of the algorithm if the numerical approximation is close to a breadth-one isolated singular solution.

By introducing some well-chosen smoothing parameters to the given system, we develop an improved deflation technique, which derives a square and regular augmented system from an isolated singular solution in a finite number of deflations. Based on this technique, we propose an algorithm for computing verified error bounds such that a slightly perturbed polynomial system is guaranteed to possess an isolated singular solution within the computed bounds.

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Title: Removable Singularities of Ore Operators
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Defended: December 2013

Ore algebras are an algebraic structure used to model many different kinds of functional equations like differential and recurrence equations. The elements of an Ore algebra are polynomials for which the multiplication is defined to be usually non-commutative. As a consequence, Gauß' lemma does not hold in many Ore polynomial rings and hence the product of two primitive Ore polynomials is not necessarily primitive. This observation leads to the distinction of non-removable and removable factors and to the study of desingularizing operators.

Desingularization is the problem of finding a left multiple of a given Ore operator in which some factor of the leading coefficient of the original operator is removed. We derive a normal form for such left factors and unify known results for differential and shift operators into one desingularization algorithm. Furthermore, we analyze the effect of removable and non-removable factors on computations with Ore operators.

The set of operators of an Ore algebra that give zero when applied to a given function forms a left ideal. The cost of computing an element of this ideal depends on the size of the coefficients (the degree) and the order of the operator. In order to be able to predict or reduce these costs, we derive an order-degree curve. For a given Ore operator, this is a curve in the (r, d)-plane such that for all points (r, d) above this curve, there exists a left multiple of order r and degree d of the given operator. We show how desingularization yields order-degree curves which are extremely accurate in examples. When computed for the generator of an operator ideal from applications like physics or combinatorics, the resulting bound is usually sharp.

The generator of a left ideal in an Ore polynomial ring is the greatest common right divisor of the ideal elements, which can be computed by the Euclidean algorithm. Polynomial remainder sequences contain the intermediate results of the Euclidean algorithm when applied to (non-)commutative polynomials. The running time of the algorithm is dependent on the size of the coefficients of the remainders. Different methods have been studied to make these as small as possible. The subresultant sequence of two polynomials is a polynomial remainder sequence in which the size of the coefficients is optimal in the generic case, but when taking the input from applications, the coefficients are often larger than necessary. We generalize two improvements of the subresultant sequence to Ore polynomials, in which we show that the non-removable factors of the greatest common right divisor appear as content. Based on this result we show how to divide out this content during the Euclidean algorithm and derive a new bound for the minimal coefficient size of the remainders. Our approach also yields a new proof for the results in the commutative case, providing a new point of view on the origin of the extraneous factors of the coefficients.