Convex Algebraic Geometry and Semidefinite Optimization

[Extended Abstract]

Pablo A. Parrilo
Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
77 Massachusetts Avenue, Room 32D-726
Cambridge, MA 02139, USA
parrilo@mit.edu
http://www.mit.edu/~parrilo

ABSTRACT

In the past decade there has been a surge of interest in algebraic approaches to optimization problems defined by multivariate polynomials. Fundamental mathematical challenges that arise in this area include understanding the structure of nonnegative polynomials, the interplay between efficiency and complexity of different representations of algebraic sets, and the development of effective algorithms. Remarkably, and perhaps unexpectedly, convexity provides a new viewpoint and a powerful framework for addressing these questions. This naturally brings us to the intersection of *algebraic geometry*, *optimization*, and *convex geometry*, with an emphasis on algorithms and computation. This emerging area has become known as *convex algebraic geometry* [1].

This tutorial will focus on basic and recent developments in convex algebraic geometry, and the associated computational methods based on semidefinite programming for optimization problems involving polynomial equations and inequalities; see e.g. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. There has been much recent progress, by combining theoretical results in real algebraic geometry with semidefinite programming to develop effective computational approaches to these problems. We will make particular emphasis on sum of squares decompositions, general duality properties, infeasibility certificates, approximation/inapproximability results, as well as survey the many exciting developments that have taken place in the last few years.

Categories and Subject Descriptors

I.1.2 [**Symbolic and algebraic manipulation**]: Algebraic algorithms; G.1.6 [**Numerical Analysis**]: Optimization—*Convex programming*

General Terms

Theory, Algorithms

Keywords

convex algebraic geometry; multivariate polynomials; semidefinite programming; convex optimization

ISSAC'13, June 26–29, 2013, Boston, Massachusetts, USA.
ACM 978-1-4503-2059-7/13/06.

1. ACKNOWLEDGMENTS

This work was partially supported by NSF grant DMS-0757207 and a Finmeccanica Career Development Chair.

2. REFERENCES

[1] G. Blekherman, P. A. Parrilo, and R. Thomas, editors. *Semidefinite optimization and convex algebraic geometry*, volume 13 of *MOS-SIAM Series on Optimization*. SIAM, 2012.

[2] K. Gatermann and P. A. Parrilo. Symmetry groups, semidefinite programs, and sums of squares. *Journal of Pure and Applied Algebra*, 192(1-3):95–128, 2004.

[3] M. Kojima, S. Kim, and H. Waki. Sparsity in sums of squares of polynomials. *Mathematical Programming*, 103(1):45–62, May 2005.

[4] J. B. Lasserre. Global optimization with polynomials and the problem of moments. *SIAM J. Optim.*, 11(3):796–817, 2001.

[5] J. Löfberg. Pre-and post-processing sum-of-squares programs in practice. *IEEE Transactions on Automatic Control*, 54(5):1007–1011, 2009.

[6] Y. Nesterov. Squared functional systems and optimization problems. In J. Frenk, C. Roos, T. Terlaky, and S. Zhang, editors, *High Performance Optimization*, pages 405–440. Kluwer Academic Publishers, 2000.

[7] P. A. Parrilo. *Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization*. PhD thesis, California Institute of Technology, May 2000.

[8] P. A. Parrilo. Semidefinite programming relaxations for semialgebraic problems. *Math. Prog.*, 96(2, Ser. B):293–320, 2003.

[9] H. Peyrl and P. Parrilo. Computing sum of squares decompositions with rational coefficients. *Theoretical Computer Science*, 409(2):269–281, 2008.

[10] S. Prajna, A. Papachristodoulou, and P. A. Parrilo. *SOSTOOLS: Sum of squares optimization toolbox for MATLAB*, 2002-05. Available from `http://www.cds.caltech.edu/sostools` and `http://www.mit.edu/~parrilo/sostools`.

[11] N. Z. Shor. Class of global minimum bounds of polynomial functions. *Cybernetics*, 23(6):731–734, 1987. (Russian orig.: Kibernetika, No. 6, (1987), 9–11).