Example

For example, use inequalities (2) and (6) defined above

$$-x_1 - 2x_2 \leq -4$$

$$6x_1 - 2x_2 \leq 17$$

We need eliminate x_1

Pick $\lambda_1 = 6$, $\lambda_2 = 1$ to give

$$6(-x_1 - 2x_2) + (6x_1 - 2x_2) \leq 6(-4) + 17$$

$$-2x_2 \leq -1$$

FM Matrix notation example

- Constructing such inequalities corresponds to multiplication of original Ax≤b by a positive matrix C

$-4x_1 - x_2 \leq -9 \ (1)$

$-x_1 - 2x_2 \leq -4 \ (2)$

$6x_1 - 2x_2 \leq 17 \ (6)$

$x_1 + 2x_2 \leq 11 \ (5)$

$$C = \begin{bmatrix} 6 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 6 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} -4 & -1 \\ -1 & -2 \\ 6 & -2 \\ 1 & 2 \end{bmatrix} \leq b = \begin{bmatrix} -9 \\ -4 \\ 17 \\ 11 \end{bmatrix}$$

The projection

$$CA = \begin{bmatrix} 0 & -14 \\ 0 & 7 \\ 0 & -10 \\ 0 & 0 \end{bmatrix} \quad Cb = \begin{bmatrix} 14 \\ 35 \\ -7 \\ 24 \end{bmatrix}$$

This gives the system of inequalities for *P(S)* as

$-14x_2 \leq 14$ $7x_2 \leq 35$ $-10x_2 \leq -7$ $0 \leq 24$

Simplified pair: $-x_2 \leq -7/10$ $x_2 \leq 5$

Equalities elimination

- If input system has both equalities and inequalities equalities are eliminated first

$$x_1 - x_2 = 2$$
$$x_1 + 3x_2 \leq 5$$
$$-2x_1 - x_2 \leq 10$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} = \\ \leq \\ \leq \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} = \\ \leq \\ \leq \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 14 \end{bmatrix}$$

Result: $4x_2 \leq 3$ $-3x_2 \leq 14$

Simplified: $-14/3 \leq x_2 \leq 3/4$